GCA TECHNICAL REPORT NO. 66-14-N

STUDIES OF ATMOSPHERIC STRUCTURE AND THE VARIABILITY OF THE EARTH'S ATMOSPHERE

R. A. MINZNER

GCA CORPORATION
Bedford, Massachusetts

FINAL REPORT

CONTRACT NO. NASW-1225

PREPARED FOR

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

HEADQUARTERS

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by

R.A. Minzner

SUMMARY

The work under Contract NASw-1225 "Studies of Atmospheres and Structures and Variability of the Earth's Atmosphere" consisted of investigations in five major areas of interest of which the first dealt with various forms of density-altitude equations.

Some previously published and often used equations are shown to have functional limitations which lead to progressively erroneous results when the temperature-altitude gradient approaches certain realistic non-zero values. Series expansion forms of these equations eliminate the difficulty but are still cumbersome and slowly converging. Other frequently used density-altitude equations having a similar functional limitation for temperature-altitude gradients approaching zero loose this limitation when these equations are transformed into simple series expansions. These series converge very rapidly so that usable results are obtained from the first term while two terms lead to five-significant figure accuracy. A comparison of computed helium number densities with values obtained by numerous direct observations suggest a variation in the onset of diffusive separation.

Another area of activity dealt with the accumulation of atmospheric density data for use in statistical analysis. Data for a total of 217 density-altitude profiles ranging between 20 and 220 km were collected, catalogued, key punched, and published in a single volume. These data do not include data collected by the Meteorological Rocket Network.

A third area of interest involved the preliminary study of some of the density data, and a comparison of these results with current atmospheric density models. Mean summer and mean winter density-altitude profiles pairs for 30°N, for 38°N, and for 58°N each show crossings or isopycnic regions near 90 km altitude. While the crossing for each pair of profiles exist at nearly the same altitude, they occur at different densities. The profile pairs for 38° and 58° each exhibit an additional isopycnic level at about two scale heights above 90 km, in accordance with a simple theory. The mean of all data shows the standard-atmosphere densities to be too low between 83 and 120 km.

A fourth area of investigation involved an updated study of a method for temperature-altitude determination using the simultaneously measured number density data for two inert gases, a light gas and a heavy gas such as helium

and argon. The method is analyzed for two nearly extreme conditions of solar activity and rigorous error analysis based on realistic measuring sensitivities indicated how the uncertainties in the temperature-altitude profiles may be minimized. The error analysis considers two methods of numerical integration, the linear tropozoidal rule and the logarithmic tropozoidal rule.

The fifth and final area of investigation involved problems relating to the preparation of the Supplementary Atmospheres 1966, a Government Printing Office Publication which is intended as a companion, document to the U.S. Standard Atmosphere 1962. The preparation of the Supplementary Atmospheres as applicable to this contract, involved the following activities:

A large number of transition models were developed each between different sets of previously specified boundary conditions (temperature and density). An exact solution, which requires the existence of an isothermal layer within the transition region, has been developed.

A particular transition model, designed as a modification to the Standard Atmosphere, reflects the higher densities observed between 80 and 120 km. The Standard Atmosphere definitions were maintained up to 69 km where the temperature gradient was reduced from -4 degrees/km to -3 degrees/km for the altitude interval from 69 to 79 km above which an isothermal layer at 190.65 degrees K extended from 79 to 90 km. This modification produced a density increase of about 13 at 90 km and produced a kind of average model in better keeping with the mean of observed data.

The lack of explicit equations for reproducing the geopotential to geometric-altitude relationships given in the Standard Atmosphere 1962 led to the development of empirical relationships which closely fit the tabular data. These relationships in turn were used to extend those Standard Atmosphere tables in terms of geopotential from about 88 geopotential kilometers to 120 kilometers. A study of the constants required in the calculation of the Standard Atmosphere pressures suggests that some small inconsistencies may have been involved in the calculation of the Standard Atmosphere. The first four areas of study have been previously published in scientific reports while this final report is devoted primarily to the fifth area of study.

INTRODUCTION

The contract for which this document serves as the final report has the rather unwieldly title of "Studies of Atmospheres and Structure and Variability of the Earth's Atmosphere:" An edited version of this contract title serving as the title of this final report indicates the scope of the work of this contract being reported upon.

The work of this contract covered five major discrete areas of endeavor which were not necessarily equally demanding of labor: (1) theoretical relationships between atmospheric properties, (2) observation of thermodynamic properties particularly the density of the Earth's atmosphere, (3) empirical and theoretical model atmospheres, (4) refinements in temperature determination methods, and (5) problems related to the preparation of Supplementary Atmospheres, 1966, a companion volume to the U.S. Standard Atmosphere 1962. The work in the first four areas has been previously reported in scientific reports under this contract and are not reviewed in the main body of the final report. Rather, four appendices are devoted to these four areas. Each successive appendix contains the title page, the summary, and the references of the respectively related scientific reports. Since the fifth area of activity has not been previously reported formally, the bulk of this final report deals with the fifth phase of the contractual effort.

BACKGROUND TO SUPPLEMENTARY ATMOSPHERES WORK

The ARDC Model Atmosphere 1956 [1]* and the closely related United States Standard Atmosphere 1958 [2] were developed from very limited atmospheric data obtained from rocket-probe instrumentation. The high-altitude portions of these models represent conditions, which, in retrospect, may be considered as characteristic of relatively low solar activity.

The advent of artificial earth satellites with perigees in the vicinity of 200 to 700 km permitted the determination of orbital deceleration resulting from the drag force of the earth's atmosphere. Atmospheric density data inferred from the earliest of these drag-acceleration observations led to the preparation of the ARDC Model Atmosphere, 1959 [3].

The data used in the preparation of the 1959 Model, plus other high-altitude density data, resulted in the revision of the 1958 United States Standard Atmosphere, particularly for altitudes above 90 km [4]. Other political and scientific considerations involving temperatures between 20 and 30 km altitude made it expedient to revise that United States Standard Atmosphere even at these low altitudes. Consequently, the combined efforts of various working groups resulted in the preparation of the United States Standard Atmosphere, 1962 [5], which differed from the 1958 Standard Atmosphere at all altitudes above 20 km. This model has been used as representative of atmospheric conditions for a midlatitude atmosphere during a period of medium-high solar activity.

The increased availability of temperature, density, and wind data to altitudes of 90 km and greater, at various seasons of the year and for various latitudes, plus a need for atmospheric models showing such seasonal and latitudinal variation, led COESA (the Committee for Extension of the Standard Atmosphere) to plan a set of such seasonal and latitudinal atmospheres which would be supplemental to the Standard Atmosphere. To this end, Cole and Kantor [6] prepared "Air Force Interim Supplemental Atmospheres to 90 km." These models were limited to altitudes below 90 km, and were not continuous with the Standard Atmosphere at 90 km.

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*Numbers in [ ] represent reference numbers.

Jacchia [7] developed a set of atmospheric models which extended upward from a set of arbitrary boundary conditions at 120 km and which reflected variations in solar radiation, geomagnetic activity, time of day, and time of the year, that is, the semiannual variation. It was decided by COESA to simply connect the Cole and Kantor Models to the Jacchia Models by a set of transition models for the proposed Supplementary Atmospheres publication. Later the scope of the goals of COESA concerning the Supplementary Atmospheres was increased, and various types of problems were presented for solution and calculation.

This report deals with the major problems related to the preparation of the Supplementary Atmosphere publication. The writer's contribution to the solution of six of these problems comprise the following six chapters of this report.

The activities under the contract also dealt with four other major areas of interest each of which has been reported in separate technical reports. These are:

(1) Critical Examination of Equations for Atmospheric Number-Density Calculations -- Computed Values Compared with Observations. [8]

(2) Density-Altitude Data from 150 Rocket Flights and 26 Searchlight Probings, 1947 through 1964. [9]

(3) A Status Report on Atmospheric Density Models and Observations. [10]

(4) Temperature Determination of Planetary Atmospheres - Optimum Boundary Conditions for Both Low and High Solar Activity. [11]

The title page, abstract or summary, table of contents, and list of references of each of these four technical reports are reproduced as four appendices to this report which serves as a final report to contract NASW-1225.

TRANSITION MODELS

Initial Requirement

A desire was expressed through COESA to the effect that any supplemental atmospheres to be published under its direction be made continuous with some model or models extending to altitudes as great as 700 or 1000 km. In particular, the supplemental atmospheres should be continuous with either the U.S. Standard Atmosphere, 1962, at 120 km altitude, or with some other reasonable model or set of models at 120 km. This choice of 120 km was related to the fact that Jacchia [7] had published a set of "Static Diffusion Models of the Upper Atmosphere . . ." for altitudes above 120 km. These models have a common set of boundary conditions at 120 km close to those of the standard atmosphere. (Jacchia's common density, 0.2461×10^{-10} gm/cm^3, is 1.02627 percent greater than that of the U.S. Standard Atmosphere, while his common temperature, 355.0°K, is 1.1730 percent smaller than that of the standard atmosphere).

Conventional Method of Calculation

Two approaches for developing transition atmospheres between the Cole-Kantor Models [6] and the common point of the Jacchia models were presented to the task group charged with the problem. One approach discussed by the committee is the conventional cut-and-try method in which base-level boundary conditions are adopted, and then a large number of trial calculations are made using various numbers of atmospheric layers with corresponding sets of temperature-altitude gradients. The number of possible combinations of values of layers and gradiants is unlimited and only in very rare instances can an exact match between the two sets of boundary conditions be made. Usually, only a close approximation to the desired values can be achieved.

Exact Calculation Method

A second method, this one presented to the task group by the writer, permits the exact connection of a realistic temperature-density pair, ρ_b and $(T_M)_b$ at geopotential altitude h_b, with a second realistic temperature-density pair, ρ_a and $(T_M)_a$ at geopotential altitude h_a, provided only that the altitude interval, $h_a - h_b$, contains an isothermal layer of unspecified extent in altitude, where the data computed downward from h_a may be matched to data computed upward from h_b.

In its simplest form, the method is applied to a situation where h_b is the base of the isothermal layer which extends upward to some unknown geopotential altitude h_x, and h_a is the upper end of the next higher layer which is characterized by a single non-zero temperature-altitude gradient extending from h_x to h_a. The density ρ_x at h_x may be expressed relative to the base of the isothermal layer by the expression;

$$\rho_x = \rho_b \exp\left\{ \frac{-(h_x - h_b)\, Q}{(T_M)_b} \right\} \tag{1}$$

where

$$Q = \frac{G\, M_o}{R} = 0.0341631947\,^{o}K/m' \tag{2}$$

and

G is the geopotential transformation coefficient equal to 9.80665 m^2 sec^{-2} $(m')^{-1}$

M is the sea-level value of the molecular weight of air, 28.9644 kilogram per kilomole (kg/kmol)

R is the universal gas constant 8.31432 x 10^3 joules $(^{o}K)^{-1}$ $(kmol)^{-1}$

The density ρ_x at h_x may also be expressed relative to the top of the constant-gradient layer by the expression

$$\rho_x = \rho_a \left[\frac{(T_M)_a}{(T_M)_b}\right]^{\left\{1 + \frac{(h_a - h_x)\, Q}{\left[(T_M)_a - (T_M)_b\right]}\right\}} \tag{3}$$

The value of h_x is found by the simultaneous solution of Equations (1) and (3) as expressed by;

$$\rho_b \exp\left\{ \frac{-(h_x - h_b)\, Q}{(T_M)_b} \right\} = \rho_a \left[\frac{(T_M)_a}{(T_M)_b}\right]^{\left\{1 + \frac{(h_a - h_x)\, Q}{(T_M)_a - (T_M)_b}\right\}} \tag{4}$$

The temperature-altitude gradient between h_x and h_a specified by $L_{x,a}$ is given by

$$L_{x,a} = \frac{(T_M)_a - (T_M)_b}{h_a - h_x} \tag{5}$$

Since no analytical solution for h_x in Equation (4) has been found, numerical methods involving digital computers must be used.

Initial Graphical and Numerical Results

The method has been previously discussed in some detail by Minzner [12] in conjunction with detailed tables of nine transition models computed to connect the Cole-Kantor models to the Standard Atmosphere at 120 km. Temperature-altitude profiles for seven of these models are shown in Figure 1 and the seven corresponding density-altitude profiles expressed as percentage departures of model densities from the density of the standard-atmosphere in Figure 2. A summary of values of the boundary conditions of all nine of these models is given in Table 1.

Later Graphical and Numerical Results

When COESA indicated that summer and winter forms of the Jacchia-type models would be developed with summer densities 20 percent lower and winter densities 50 percent greater than the standard-atmosphere densities at 120 km, the exact method of calculation was again used to develop appropriate sets of transition models between the summer and winter Cole-Kantor models and the appropriate revised 120-km boundary conditions. These results are shown in Figures 3 and 4 and the summary of the boundary conditions are shown in Table 2. In Tables 3 through 10, the detailed numerical values for these summer and winter transition models are given. (The 120-km values of temperature are identical for all seven cases computed, since, at the time of these calculations, the determination of the summer and winter temperature for the base of the revised Jacchia-type models had not yet been determined by the committee. Since these calculations had been made, however, the committee has fixed upon 355.90°K for the summer temperature at 120 km and 410.90°K for the winter temperature at that altitude.) The lack of an accurate temperature at 120 km does not seriously influence the densities at lower altitudes, but an inaccurate value of density at 120 km would have seriously influenced the temperatures at lower altitudes. The difference between summer densities and winter densities at 120 km in these models is the principal reason for the relatively large spread between that cluster of data points (h_x, T_{Mx}) for summer models on Figure 3 and that cluster for winter models in the same figure.

The sharp breaks in the various curves at 110 km in Figure 4 indicate that the standard atmosphere has an abrupt change in temperature gradient at that altitude, and consequently an abrupt change in the slope of the density-altitude curve at that altitude. The various models Ar through Gr, on the other hand, all have constant temperature-altitude gradients between about 104 km altitude to 120 km altitude and consequently have no sharp change in slope in the density-altitude profile in that region. The difference curves however, between each of these smooth density-altitude profiles, (above 104 km) discontinuity at 110 km, are seen as curves with a slope discontinuity at 110 km in Figure 4. A similar explanation applies to the slope discontinuities at 90 km, and if the graph had been plotted with greater resolution, another set of small slope discontinuities would be seen at Z = 100 km. The very prominent discontinuities at altitudes 94.78, 96.41, and 97.58 km

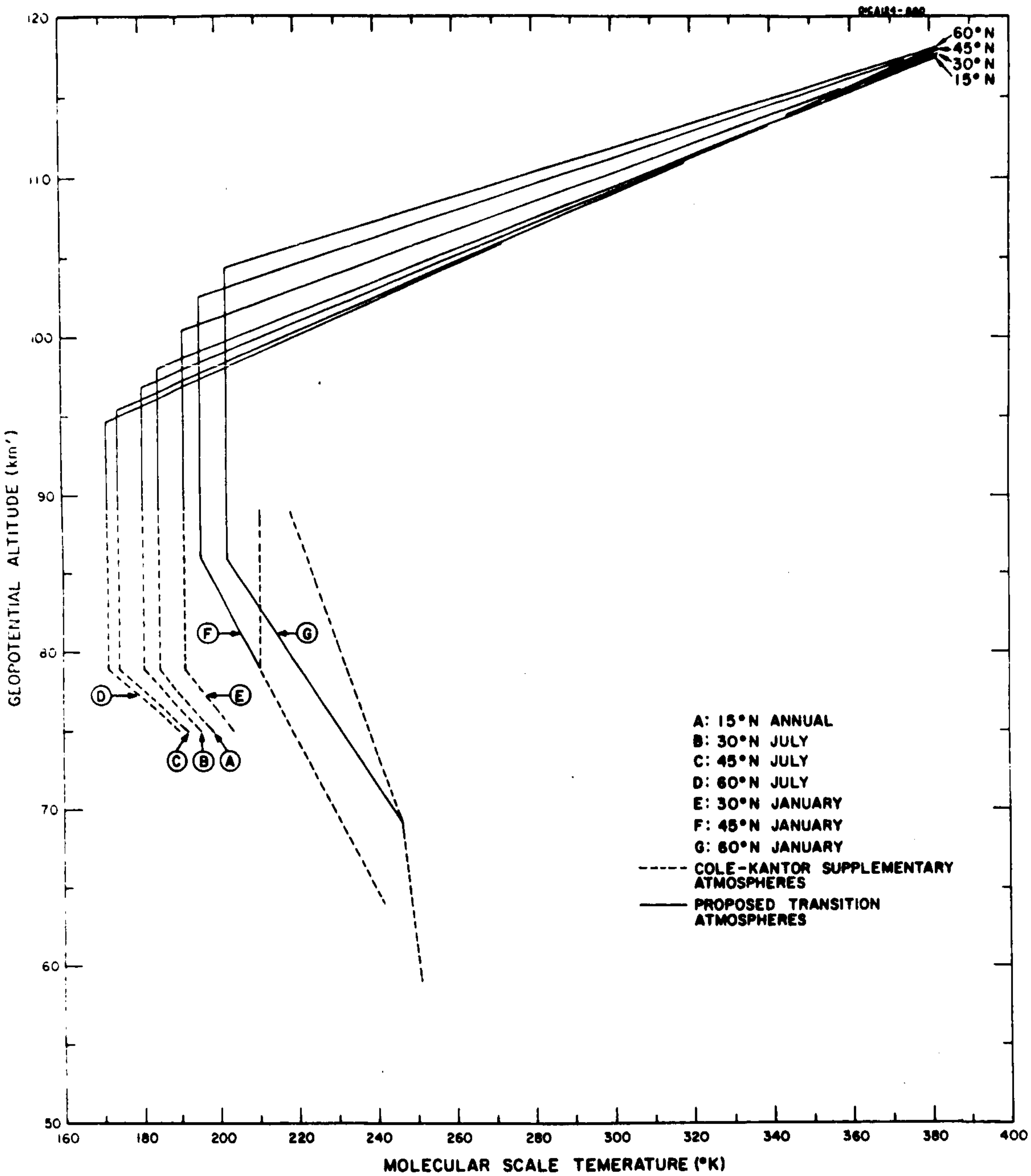

Figure 1. Temperature-altitude profiles defining seven proposed transition models connecting supplementary atmospheres to thermospheric models.

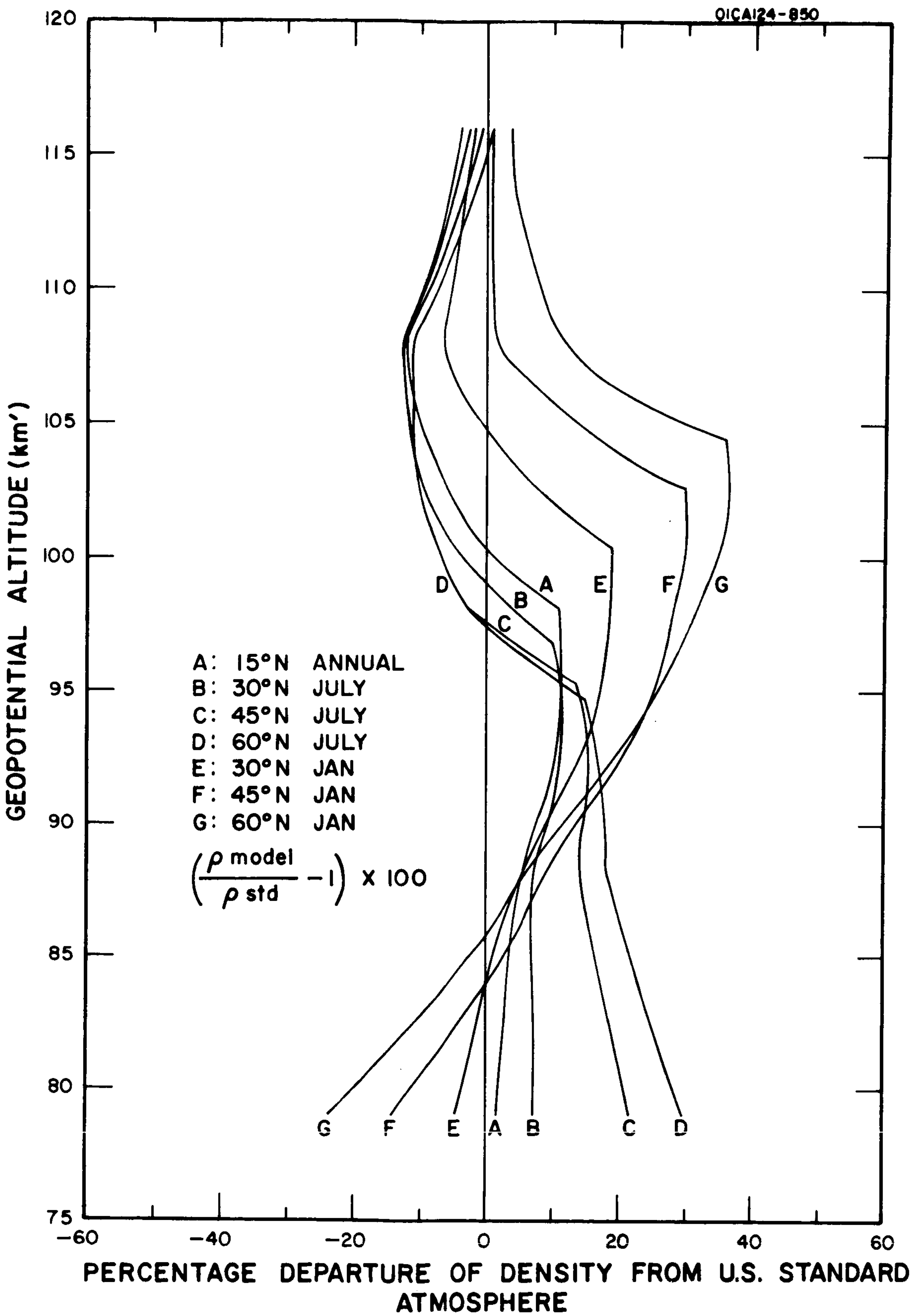

Figure 2. Percentage departure of the density of various models from that of the U. S. Standard Atmosphere versus geopotential altitude.

TABLE 1

BOUNDARY-CONDITION VALUES WHICH WERE USED IN DEDUCING THE VALUE OF W_X AND THE POSITIVE TEMPERATURE-ALTITUDE GRADIENT FOR EACH OF THE TRANSITION MODELS A THROUGH I

		h_{bb} km′	ρ_{bb} gm/m^3	$(T_M)_{bb}$ °K	L_{bb} °K/km′	h_b km′	ρ_b gm/m^3	$(T_M)_b$ °K	H'_b km′	h_a km′	ρ_a gm/m^3	$(T_M)_a$ °K	H'_a km′
A	15°N Annual					79	0.20265×10^{-1}	184.15	5.3903031	117.4958	0.24610×10^{-4}	382.244	11.18876
B	30°N July					79	0.21523×10^{-1}	180.15	5.2732180	117.6118	0.24610×10^{-4}	382.244	11.18876
C	45°N July					79	0.24315×10^{-1}	174.15	5.0975904	117.7765	0.24610×10^{-4}	382.224	11.18876
D	60°N July					79	0.25997×10^{-1}	171.15	5.0097767	117.9299	0.24610×10^{-4}	382.244	11.18876
E	30°N Jan					79	0.19096×10^{-1}	191.15	5.5952019	117.6118	0.24610×10^{-4}	382.244	11.18876
F	45°N Jan	64	0.14361	241.65	-2.1	86	0.562606×10^{-2}	195.45	5.7210682	117.7765	0.24610×10^{-4}	382.244	11.18876
F′	45°N Jan alt.	52	0.65845	265.65	-4.2	85	0.651826×10^{-2}	196.35	5.7474122	117.7765	0.24610×10^{-4}	382.244	11.18876
G	60°N Jan	69	0.59480×10^{-1}	246.15	-2.6	86	0.53815×10^{-2}	201.95	5.9113313	117.9299	0.24610×10^{-4}	382.244	11.18876
H	60°N Cold	71	0.34111×10^{-1}	255.15	-3.6	90	0.24111×10^{-2}	186.75	5.4664082	117.9299	0.24610×10^{-4}	382.244	11.18876
I	60°N Warm	54	0.54953	268.15	-2.7	79	0.18725×10^{-1}	200.65	5.8732787	117.9299	0.24610×10^{-4}	382.244	11.18876

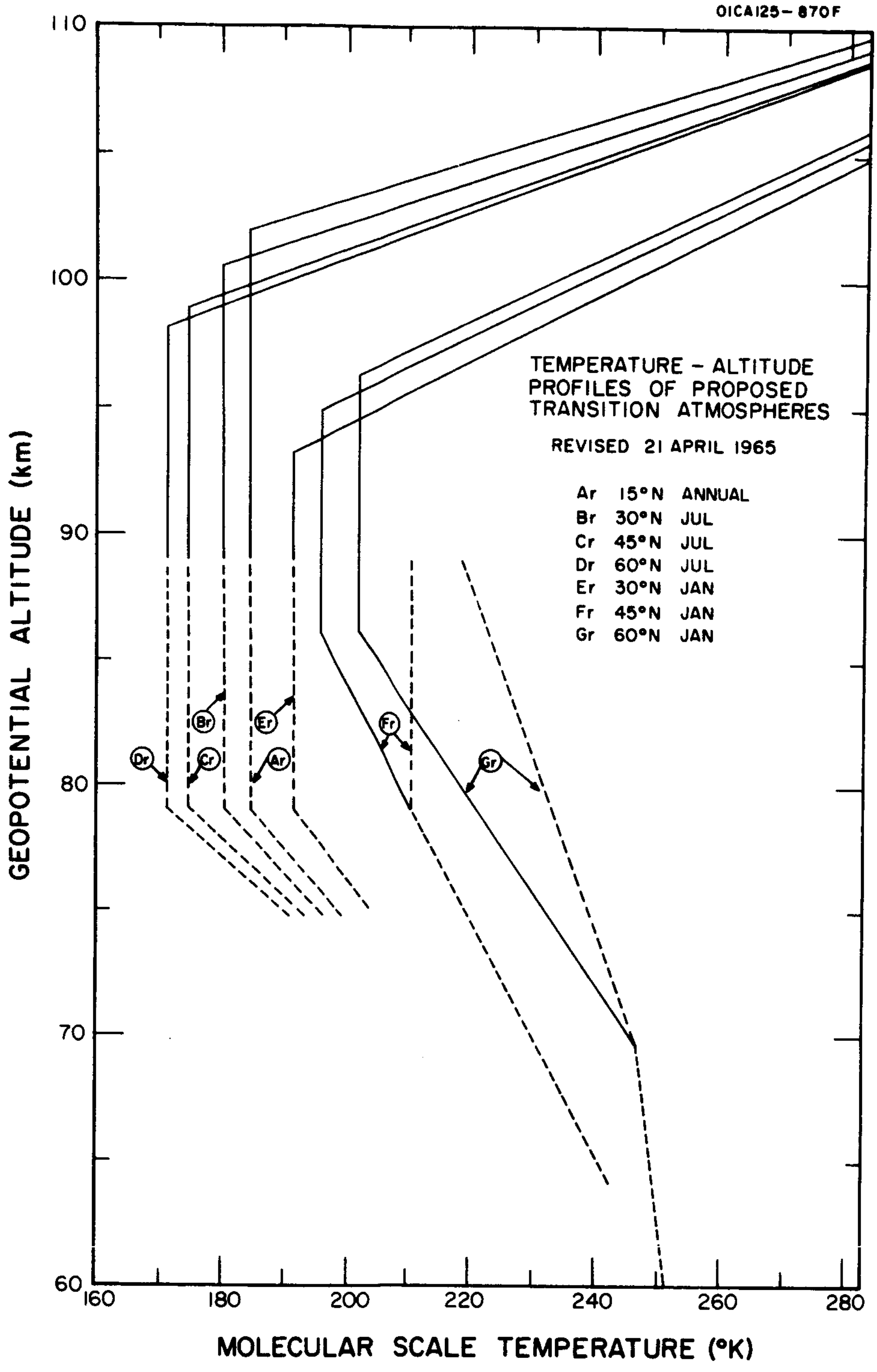

Figure 3. Defining temperature-altitude profiles for seven proposed transition models connecting supplementary atmospheres to thermosphere models.

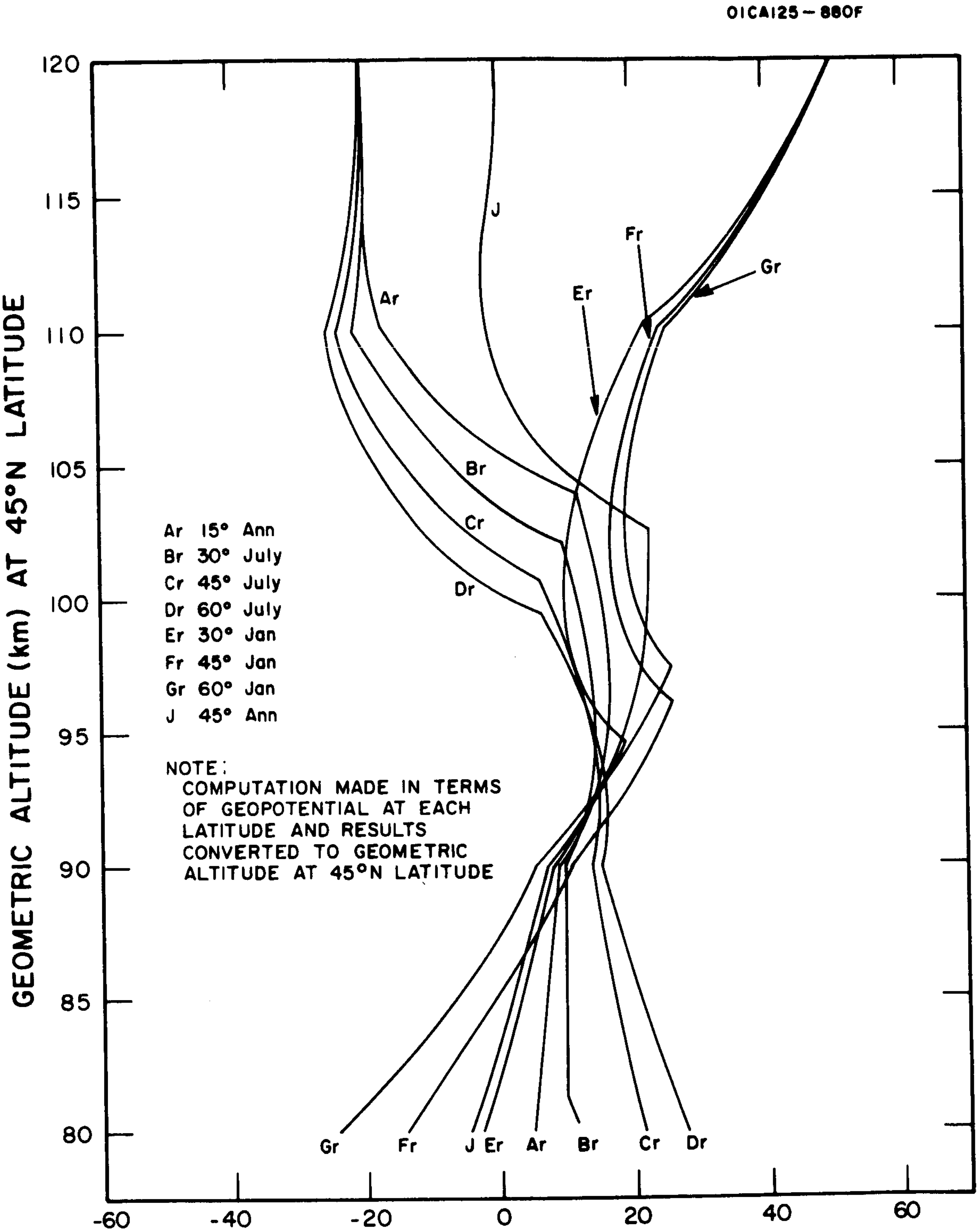

Figure 4. Percentage departure of the density of revised transition models from that of the U.S. Standard Atmosphere versus geometric altitude.

TABLE 2

BOUNDARY-CONDITION VALUES WHICH WERE USED IN DEDUCING THE VALUE OF h_x AND THE POSITIVE TEMPERATURE-ALTITUDE GRADIENT FOR EACH OF THE TRANSITION MODELS Ar THROUGH Ir

		h_{bb} km′	ρ_{bb} gm/m^3	$(T_M)_{bb}$ °K	L_{bb} °K/km′	h_b km′	ρ_b gm/m^3	$(T_M)_b$ °K	H_b' km′	h_a km′	ρ_a gm/m^3	$(T_M)_a$ °K	H_a' km′
Ar	15°N Annual					79	0.202650E-01	184.15	5.3903031	117.4958	0.19489E-04	382.244	11.18876
Br	30°N July					79	0.215230E-01	180.15	5.2732180	117.6118	0.19489E-04	382.244	11.18876
Cr	45°N July					79	0.243150E-01	174.15	5.0975904	117.7765	0.19489E-04	382.244	11.18876
Dr	60°N July					79	0.259970E-01	171.15	5.0097767	117.9299	0.19489E-04	382.244	11.18876
Er	30°N Jan.					79	0.190960E-01	191.15	5.5952019	117.6118	0.36539E-04	382.244	11.18876
Fr	45°N Jan.	64	0.14561E-00	241.65	-2.1	86	0.562606E-02	195.45	5.7210682	117.7765	0.36539E-04	382.244	11.18876
F′r	45°N Jan.	52	0.65845E-00	265.65	-4.2	85	0.651826E-02	196.35	5.7474122	117.7765	0.36539E-04	382.244	11.18876
Gr	60°N Jan.	69	0.59480E-01	246.15	-2.6	86	0.538150E-02	201.95	5.9113313	117.9299	0.36539E-04	382.244	11.18876

TABLE 3*

MODEL AR

PROPERTIES OF TRANSITION ATMOSPHERE
TROPICAL (15N) ANN TO JACCHIA MODELS

15N ANN X = 101.96380 KM DEN X = .28612951E-03 G/M3
GEOPOT SCALE HT AT 117.4958 KM = 11.18876, AT X KM = 5.3903031
GRADIENT OF GEOP SCALE HT ABOVE X = .37332327E 00
GRADIENT OF MOL. TEMP. ZERO FOR H BTWN 79. AND X
GRADIENT OF MOL. TEMP. 12.75391 DEG/KM FOR H BTWN X AND 117.4958

ALTITUDE KM		TEMP DEG KELVIN		MOL WT	DENSITY	SCALE HT KM	
GEOMET	GEOPOT	MOL.	KINETIC		G/M3	GEOMET	GEOPOT
80.186	79.000	184.15	184.15	28.96	.20265E-01	5.540	5.390
81.214	80.000	184.15	184.15	28.96	.16833E-01	5.542	5.390
82.000	80.764	184.15	184.15	28.96	.14608E-01	5.544	5.390
83.271	82.000	184.15	184.15	28.96	.11615E-01	5.546	5.390
84.000	82.708	184.15	184.15	28.96	.10185E-01	5.547	5.390
85.329	84.000	184.15	184.15	28.96	.80149E-02	5.549	5.390
86.000	84.651	184.15	184.15	28.96	.71030E-02	5.551	5.390
87.389	86.000	184.15	184.15	28.96	.55304E-02	5.553	5.390
88.000	86.592	184.15	184.15	28.96	.49545E-02	5.554	5.390
89.450	88.000	184.15	184.15	28.96	.38161E-02	5.556	5.390
90.000	88.533	184.15	184.15	28.96	.34567E-02	5.557	5.390
91.512	90.000	184.15	183.49	28.86	.26331E-02	5.560	5.390
92.000	90.472	184.15	183.27	28.83	.24122E-02	5.561	5.390
93.576	92.000	184.15	182.59	28.72	.18169E-02	5.564	5.390
94.000	92.410	184.15	182.40	28.69	.16837E-02	5.564	5.390
95.641	94.000	184.15	181.68	28.58	.12537E-02	5.567	5.390
96.000	94.347	184.15	181.52	28.55	.11754E-02	5.568	5.390
97.707	96.000	184.15	180.78	28.43	.86509E-03	5.571	5.390
98.000	96.282	184.15	180.65	28.41	.82085E-03	5.571	5.390
99.775	98.000	184.15	179.87	28.29	.59693E-03	5.574	5.390
100.000	98.217	184.15	179.77	28.28	.57333E-03	5.575	5.390
101.844	100.000	184.15	178.97	28.15	.41189E-03	5.578	5.390
102.000	100.150	184.15	178.90	28.14	.40054E-03	5.578	5.390
103.876	101.963	184.15	178.08	28.01	.28612E-03	5.581	5.390
103.914	102.000	184.61	178.51	28.01	.28350E-03	5.596	5.404
104.000	102.082	185.67	179.49	28.00	.27762E-03	5.628	5.435
105.986	104.000	210.12	202.14	27.86	.17611E-03	6.373	6.150
106.000	104.013	210.29	202.30	27.86	.17558E-03	6.378	6.156
108.000	105.943	234.90	224.86	27.73	.11686E-03	7.129	6.876
108.058	106.000	235.63	225.52	27.72	.11554E-03	7.151	6.897
110.000	107.871	259.50	247.17	27.59	.81019E-04	7.880	7.596
110.133	108.000	261.14	248.64	27.58	.79165E-04	7.930	7.644
112.000	109.798	284.08	269.23	27.45	.58077E-04	8.632	8.315
112.208	110.000	286.64	271.52	27.44	.56187E-04	8.710	8.390
114.000	111.724	308.64	291.04	27.31	.42806E-04	9.384	9.034
114.285	112.000	312.15	294.14	27.29	.41062E-04	9.492	9.137
116.000	113.649	333.19	312.61	27.18	.32302E-04	10.137	9.753
116.364	114.000	337.66	316.51	27.15	.30757E-04	10.274	9.884
118.000	115.573	357.73	333.93	27.04	.24872E-04	10.890	10.471
118.443	116.000	363.17	338.62	27.01	.23529E-04	11.057	10.630
120.000	117.495	382.24	355.00	26.90	.19489E-04	11.644	11.189

*Read X in heading as h_x

TABLE 4*

MODEL BR

PROPERTIES OF TRANSITION ATMOSPHERE
SUBTROP (30N) JULY TO JACCHIA MODELS

30N JULY X = 100.52278 KM DEN X = .36334419E-03 G/M3
GEOPOT SCALE HT AT 117.6118 KM = 11.18876, AT X KM = 5.2732180
GRADIENT OF GEOP SCALE HT ABOVE X = .34616040E 00
GRADIENT OF MOL. TEMP. ZERO FOR H BTWN 79. AND X
GRADIENT OF MOL. TEMP. 11.82594 DEG/KM FOR H BTWN X AND 117.6118

ALTITUDE KM		TEMP DEG KELVIN		MOL WT	DENSITY	SCALE HT KM	
GEOMET	GEOPOT	MOL.	KINETIC		G/M3	GEOMET	GEOPOT
80.106	79.000	180.15	180.15	28.96	.21523E-01	5.415	5.273
81.133	80.000	180.15	180.15	28.96	.17805E-01	5.416	5.273
82.000	80.843	180.15	180.15	28.96	.15173E-01	5.418	5.273
83.188	82.000	180.15	180.15	28.96	.12185E-01	5.420	5.273
84.000	82.789	180.15	180.15	28.96	.10491E-01	5.421	5.273
85.244	84.000	180.15	180.15	28.96	.83389E-02	5.423	5.273
86.000	84.734	180.15	180.15	28.96	.72553E-02	5.425	5.273
87.302	86.000	180.15	180.15	28.96	.57068E-02	5.427	5.273
88.000	86.677	180.15	180.15	28.96	.50186E-02	5.428	5.273
89.361	88.000	180.15	180.15	28.96	.39054E-02	5.430	5.273
90.000	88.620	180.15	180.15	28.96	.34722E-02	5.431	5.273
91.421	90.000	180.15	179.54	28.87	.26727E-02	5.434	5.273
92.000	90.561	180.15	179.29	28.83	.24029E-02	5.435	5.273
93.483	92.000	180.15	178.66	28.72	.18291E-02	5.437	5.273
94.000	92.501	180.15	178.44	28.69	.16632E-02	5.438	5.273
95.546	94.000	180.15	177.78	28.58	.12517E-02	5.441	5.273
96.000	94.439	180.15	177.58	28.55	.11515E-02	5.441	5.273
97.610	96.000	180.15	176.89	28.44	.85665E-03	5.444	5.273
98.000	96.377	180.15	176.73	28.41	.79746E-03	5.445	5.273
99.675	98.000	180.15	176.01	28.30	.58626E-03	5.448	5.273
100.000	98.313	180.15	175.87	28.28	.55237E-03	5.448	5.273
101.742	100.000	180.15	175.12	28.16	.40121E-03	5.451	5.273
102.000	100.249	180.15	175.01	28.14	.38269E-03	5.452	5.273
102.282	100.522	180.15	174.89	28.12	.36334E-03	5.452	5.273
103.810	102.000	197.62	191.14	28.01	.25351E-03	5.984	5.785
104.000	102.183	199.78	193.14	28.00	.24299E-03	6.049	5.848
105.880	104.000	221.27	212.92	27.87	.16333E-03	6.704	6.477
106.000	104.115	222.64	214.18	27.86	.15946E-03	6.746	6.517
107.950	106.000	244.92	234.48	27.73	.11004E-03	7.425	7.169
108.000	106.047	245.48	234.99	27.73	.10906E-03	7.442	7.186
110.000	107.977	268.31	255.56	27.59	.77183E-04	8.140	7.854
110.022	108.000	268.58	255.80	27.59	.76890E-04	8.148	7.862
112.000	109.906	291.13	275.91	27.45	.56194E-04	8.837	8.522
112.096	110.000	292.23	276.89	27.44	.55376E-04	8.871	8.554
114.000	111.834	313.93	296.03	27.31	.41913E-04	9.535	9.189
114.171	112.000	315.88	297.74	27.30	.40915E-04	9.595	9.246
116.000	113.761	336.71	315.91	27.18	.31915E-04	10.234	9.856
116.247	114.000	339.53	318.36	27.16	.30898E-04	10.320	9.938
118.000	115.687	359.49	335.57	27.04	.24744E-04	10.933	10.523
118.324	116.000	363.18	338.74	27.02	.23779E-04	11.046	10.631
120.000	117.611	382.24	355.00	26.90	.19490E-04	11.632	11.189

*Read X in heading as h_x.

TABLE 5*

MODEL CR

PROPERTIES OF TRANSITION ATMOSPHERE
MIDLAT (45N) JULY TO JACCHIA MODELS

45N JULY X = 98.94099 KM DEN X = .48638552E-03 G/M3
GEOPOT SCALE HT AT 117.7765 KM = 11.18876, AT X KM = 5.0975904
GRADIENT OF GEOP SCALE HT ABOVE X = .32338758E 00
GRADIENT OF MOL. TEMP. ZERO FOR H BTWN 79. AND X
GRADIENT OF MOL. TEMP. 11.04795 DEG/KM FOR H BTWN X AND 117.7765

ALTITUDE KM		TEMP DEG KELVIN		MOL WT	DENSITY	SCALE HT KM	
GEOMET	GEOPOT	MOL.	KINETIC		G/M3	GEOMET	GEOPOT
79.994	79.000	174.15	174.15	28.96	.24315E-01	5.227	5.098
80.000	79.005	174.15	174.15	28.96	.24288E-01	5.227	5.098
81.019	80.000	174.15	174.15	28.96	.19983E-01	5.228	5.098
82.000	80.955	174.15	174.15	28.96	.16567E-01	5.230	5.098
83.071	82.000	174.15	174.15	28.96	.13498E-01	5.232	5.098
84.000	82.904	174.15	174.15	28.96	.11304E-01	5.233	5.098
85.124	84.000	174.15	174.15	28.96	.91178E-02	5.235	5.098
86.000	84.851	174.15	174.15	28.96	.77145E-02	5.236	5.098
87.179	86.000	174.15	174.15	28.96	.61588E-02	5.238	5.098
88.000	86.798	174.15	174.15	28.96	.52660E-02	5.240	5.098
89.235	88.000	174.15	174.15	28.96	.41601E-02	5.242	5.098
90.000	88.743	174.15	174.15	28.96	.35955E-02	5.243	5.098
91.292	90.000	174.15	173.62	28.88	.28100E-02	5.245	5.098
92.000	90.687	174.15	173.32	28.83	.24555E-02	5.246	5.098
93.351	92.000	174.15	172.76	28.73	.18981E-02	5.248	5.098
94.000	92.630	174.15	172.50	28.69	.16774E-02	5.249	5.098
95.410	94.000	174.15	171.91	28.59	.12821E-02	5.252	5.098
96.000	94.571	174.15	171.67	28.55	.11461E-02	5.253	5.098
97.472	96.000	174.15	171.06	28.45	.86604E-03	5.255	5.098
98.000	96.512	174.15	170.84	28.41	.78328E-03	5.256	5.098
99.534	98.000	174.15	170.21	28.31	.58499E-03	5.258	5.098
100.000	98.451	174.15	170.01	28.28	.53544E-03	5.259	5.098
100.505	98.940	174.15	169.80	28.24	.48638E-03	5.260	5.098
101.598	100.000	185.85	180.73	28.17	.37275E-03	5.615	5.440
102.000	100.389	190.15	184.73	28.14	.33945E-03	5.746	5.566
103.663	102.000	207.95	201.20	28.02	.23537E-03	6.287	6.087
104.000	102.325	211.55	204.51	28.00	.21941E-03	6.396	6.192
105.729	104.000	230.04	221.44	27.88	.15570E-03	6.959	6.734
106.000	104.261	232.93	224.07	27.86	.14795E-03	7.047	6.818
107.797	106.000	252.14	241.48	27.74	.10697E-03	7.633	7.380
108.000	106.195	254.30	243.42	27.73	.10330E-03	7.699	7.444
109.866	108.000	274.23	261.29	27.60	.75857E-04	8.307	8.027
110.000	108.128	275.66	262.56	27.59	.74268E-04	8.350	8.069
111.937	110.000	296.33	280.89	27.45	.55242E-04	8.982	8.674
112.000	110.060	297.00	281.48	27.45	.54734E-04	9.003	8.694
114.000	111.991	318.33	300.18	27.31	.41208E-04	9.655	9.318
114.008	112.000	318.43	300.26	27.31	.41158E-04	9.658	9.321
116.000	113.921	339.65	318.67	27.18	.31607E-04	10.308	9.942
116.081	114.000	340.52	319.42	27.17	.31277E-04	10.335	9.967
118.000	115.849	360.95	336.94	27.04	.24641E-04	10.961	10.566
118.156	116.000	362.62	338.36	27.03	.24182E-04	11.013	10.614
120.000	117.776	382.24	355.00	26.90	.19489E-04	11.615	11.189

*Read X in heading as h_x.

TABLE 6*

MODEL DR

PROPERTIES OF TRANSITION ATMOSPHERE
SUBARCTIC (60N) JULY TO JACCHIA MODELS

60N JULY X = 98.11551 KM DEN X = .57254684E-03 G/M3
GEOPOT SCALE HT AT 117.9299 KM = 11.18876, AT X KM = 5.0097767
GRADIENT OF GEOP SCALE HT ABOVE X = .31184321E 00
GRADIENT OF MOL. TEMP. ZERO FOR H BTWN 79. AND X
GRADIENT OF MOL. TEMP. 10.65356 DEG/KM FOR H BTWN X AND 117.9299

ALTITUDE KM		TEMP DEG KELVIN		MOL WT	DENSITY	SCALE HT KM	
GEOMET	GEOPOT	MOL.	KINETIC		G/M3	GEOMET	GEOPOT
79.889	79.000	171.15	171.15	28.96	.25997E-01	5.130	5.010
80.000	79.107	171.15	171.15	28.96	.25444E-01	5.130	5.010
80.913	80.000	171.15	171.15	28.96	.21292E-01	5.131	5.010
82.000	81.060	171.15	171.15	28.96	.17231E-01	5.133	5.010
82.963	82.000	171.15	171.15	28.96	.14284E-01	5.135	5.010
84.000	83.011	171.15	171.15	28.96	.11672E-01	5.136	5.010
85.013	84.000	171.15	171.15	28.96	.95824E-02	5.138	5.010
86.000	84.961	171.15	171.15	28.96	.79087E-02	5.139	5.010
87.065	86.000	171.15	171.15	28.96	.64283E-02	5.141	5.010
88.000	86.910	171.15	171.15	28.96	.53599E-02	5.143	5.010
89.118	88.000	171.15	171.15	28.96	.43123E-02	5.144	5.010
90.000	88.858	171.15	171.15	28.96	.36333E-02	5.146	5.010
91.172	90.000	171.15	170.67	28.88	.28929E-02	5.148	5.010
92.000	90.804	171.15	170.34	28.83	.24635E-02	5.149	5.010
93.228	92.000	171.15	169.84	28.74	.19407E-02	5.151	5.010
94.000	92.750	171.15	169.52	28.69	.16708E-02	5.152	5.010
95.285	94.000	171.15	169.00	28.60	.13019E-02	5.154	5.010
96.000	94.694	171.15	168.71	28.55	.11334E-02	5.155	5.010
97.344	96.000	171.15	168.16	28.46	.87338E-03	5.158	5.010
98.000	96.637	171.15	167.90	28.41	.76908E-03	5.159	5.010
99.403	98.000	171.15	167.33	28.32	.58590E-03	5.161	5.010
99.522	98.115	171.15	167.28	28.31	.57254E-03	5.161	5.010
100.000	98.578	176.09	171.90	28.28	.50800E-03	5.311	5.154
101.464	100.000	191.23	186.02	28.18	.35905E-03	5.770	5.597
102.000	100.519	196.76	191.15	28.14	.31846E-03	5.938	5.759
103.526	102.000	212.53	205.70	28.03	.23023E-03	6.417	6.221
104.000	102.458	217.42	210.19	28.00	.20923E-03	6.565	6.364
105.590	104.000	233.84	225.18	27.89	.15403E-03	7.065	6.845
106.000	104.396	238.07	229.02	27.86	.14285E-03	7.193	6.969
107.655	106.000	255.15	244.45	27.75	.10673E-03	7.713	7.469
108.000	106.333	258.70	247.64	27.73	.10069E-03	7.822	7.573
109.721	108.000	276.46	263.50	27.61	.76167E-04	8.363	8.092
110.000	108.269	279.32	266.05	27.59	.72930E-04	8.450	8.176
111.789	110.000	297.76	282.35	27.47	.55734E-04	9.013	8.716
112.000	110.203	299.93	284.26	27.45	.54056E-04	9.079	8.779
113.858	112.000	319.07	300.98	27.32	.41672E-04	9.664	9.340
114.000	112.137	320.53	302.25	27.31	.40879E-04	9.709	9.382
115.928	114.000	340.38	319.41	27.18	.31750E-04	10.316	9.963
116.000	114.069	341.11	320.04	27.18	.31462E-04	10.339	9.985
117.999	116.000	361.68	337.62	27.04	.24593E-04	10.969	10.587
118.000	116.000	361.68	337.62	27.04	.24593E-04	10.969	10.587
120.000	117.929	382.24	355.00	26.90	.19490E-04	11.600	11.189

* Read X in heading as h_x.

TABLE 7*

MODEL ER

PROPERTIES OF TRANSITION ATMOSPHERE
SUBTROP (30N) JAN TO JACCHIA MODELS

30N JAN X = 93.26377 KM DEN X = .14921116E-02 G/M3
GEOPOT SCALE HT AT 117.6118 KM = 11.18876, AT X KM = 5.5952019
GRADIENT OF GEOP SCALE HT ABOVE X = .22973350E 00
GRADIENT OF MOL. TEMP. ZERO FOR H BTWN 79. AND X
GRADIENT OF MOL. TEMP. 7.84843 DEG/KM FOR H BTWN X AND 117.6118

ALTITUDE KM		TEMP DEG KELVIN		MOL WT	DENSITY	SCALE HT KM	
GEOMET	GEOPOT	MOL.	KINETIC		G/M3	GEOMET	GEOPOT
80.106	79.000	191.15	191.15	28.96	.19096E-01	5.745	5.595
81.133	80.000	191.15	191.15	28.96	.15970E-01	5.747	5.595
82.000	80.843	191.15	191.15	28.96	.13736E-01	5.749	5.595
83.188	82.000	191.15	191.15	28.96	.11170E-01	5.751	5.595
84.000	82.789	191.15	191.15	28.96	.97012E-02	5.752	5.595
85.244	84.000	191.15	191.15	28.96	.78135E-02	5.754	5.595
86.000	84.734	191.15	191.15	28.96	.68528E-02	5.756	5.595
87.302	86.000	191.15	191.15	28.96	.54652E-02	5.758	5.595
88.000	86.677	191.15	191.15	28.96	.48418E-02	5.759	5.595
89.361	88.000	191.15	191.15	28.96	.38227E-02	5.762	5.595
90.000	88.620	191.15	191.15	28.96	.34217E-02	5.763	5.595
91.421	90.000	191.15	190.50	28.87	.26738E-02	5.765	5.595
92.000	90.561	191.15	190.24	28.83	.24186E-02	5.767	5.595
93.483	92.000	191.15	189.57	28.72	.18702E-02	5.769	5.595
94.000	92.501	191.15	189.33	28.69	.17099E-02	5.770	5.595
94.786	93.263	191.15	188.98	28.64	.14921E-02	5.771	5.595
95.546	94.000	196.93	194.33	28.58	.12722E-02	5.947	5.764
96.000	94.439	200.38	197.53	28.55	.11591E-02	6.053	5.865
97.610	96.000	212.63	208.78	28.44	.84388E-03	6.426	6.224
98.000	96.377	215.59	211.49	28.41	.78362E-03	6.516	6.311
99.675	98.000	228.32	223.07	28.30	.57637E-03	6.904	6.683
100.000	98.313	230.79	225.30	28.28	.54418E-03	6.980	6.755
101.742	100.000	244.02	237.21	28.16	.40377E-03	7.384	7.143
102.000	100.249	245.97	238.96	28.14	.38688E-03	7.443	7.200
103.810	102.000	259.72	251.19	28.01	.28920E-03	7.864	7.602
104.000	102.183	261.15	252.47	28.00	.28078E-03	7.908	7.644
105.880	104.000	275.41	265.02	27.87	.21124E-03	8.344	8.062
106.000	104.115	276.32	265.82	27.86	.20755E-03	8.372	8.088
107.950	106.000	291.11	278.69	27.73	.15701E-03	8.826	8.521
108.000	106.047	291.48	279.02	27.73	.15593E-03	8.837	8.532
110.000	107.977	306.63	292.06	27.59	.11889E-03	9.302	8.976
110.022	108.000	306.81	292.21	27.59	.11853E-03	9.307	8.981
112.000	109.906	321.77	304.95	27.45	.91857E-04	9.767	9.419
112.096	110.000	322.50	305.57	27.44	.90749E-04	9.790	9.440
114.000	111.834	336.90	317.69	27.31	.71826E-04	10.233	9.862
114.171	112.000	338.20	318.78	27.30	.70366E-04	10.273	9.900
116.000	113.761	352.03	330.28	27.18	.56781E-04	10.699	10.304
116.247	114.000	353.90	331.83	27.16	.55194E-04	10.757	10.359
118.000	115.687	367.14	342.72	27.04	.45340E-04	11.165	10.747
118.324	116.000	369.59	344.72	27.02	.43752E-04	11.241	10.818
120.000	117.611	382.24	355.00	26.90	.36540E-04	11.632	11.189

*Read X in heading as h_x.

TABLE 8*

MODEL FR

PROPERTIES OF TRANSITION ATMOSPHERE
MIDLAT (45N) JAN TO JACCHIA MODELS

45N JAN X = 94.97564 KM DEN X = .11717792E-02 G/M3
GEOPOT SCALE HT AT 117.7765 KM = 11.18876, AT X KM = 5.7210682
GRADIENT OF GEOP SCALE HT ABOVE X = .23980201E 00
GRADIENT OF MOL. TEMP. -2.1 FOR H BTWN 64. TO 86.
GRADIENT OF MOL. TEMP. ZERO FOR H BTWN 86. AND X
GRADIENT OF MOL. TEMP. 8.19240 DEG/KM FOR H BTWN X AND 117.7765

ALTITUDE KM GEOMET	ALTITUDE KM GEOPOT	TEMP DEG KELVIN MOL.	TEMP DEG KELVIN KINETIC	MOL WT	DENSITY G/M3	SCALE HT KM GEOMET	SCALE HT KM GEOPOT
79.994	79.000	210.15	210.15	28.96	.17023E-01	6.307	6.151
80.000	79.005	210.14	210.14	28.96	.17009E-01	6.307	6.151
81.019	80.000	208.05	208.05	28.96	.14603E-01	6.246	6.090
82.000	80.955	206.04	206.04	28.96	.12594E-01	6.188	6.031
83.071	82.000	203.85	203.85	28.96	.10696E-01	6.124	5.967
84.000	82.904	201.95	201.95	28.96	.92717E-02	6.069	5.911
85.124	84.000	199.65	199.65	28.96	.77836E-02	6.002	5.844
86.000	84.851	197.86	197.86	28.96	.67841E-02	5.949	5.792
87.179	86.000	195.45	195.45	28.96	.56260E-02	5.879	5.721
88.000	86.798	195.45	195.45	28.96	.48932E-02	5.881	5.721
89.235	88.000	195.45	195.45	28.96	.39662E-02	5.883	5.721
90.000	88.743	195.45	195.45	28.96	.34828E-02	5.884	5.721
91.292	90.000	195.45	194.85	28.88	.27961E-02	5.887	5.721
92.000	90.687	195.45	194.52	28.83	.24795E-02	5.888	5.721
93.351	92.000	195.45	193.89	28.73	.19712E-02	5.890	5.721
94.000	92.630	195.45	193.59	28.69	.17656E-02	5.892	5.721
95.410	94.000	195.45	192.94	28.59	.13896E-02	5.894	5.721
96.000	94.571	195.45	192.66	28.55	.12575E-02	5.895	5.721
96.416	94.975	195.45	192.47	28.52	.11717E-02	5.896	5.721
97.472	96.000	203.84	200.22	28.45	.94286E-03	6.151	5.967
98.000	96.512	208.04	204.08	28.41	.84862E-03	6.279	6.089
99.534	98.000	220.23	215.24	28.31	.63220E-03	6.650	6.446
100.000	98.451	223.92	218.60	28.28	.58007E-03	6.762	6.554
101.598	100.000	236.61	230.09	28.17	.43623E-03	7.149	6.926
102.000	100.389	239.80	232.96	28.14	.40708E-03	7.246	7.019
103.663	102.000	253.00	244.78	28.02	.30859E-03	7.649	7.406
104.000	102.325	255.67	247.16	28.00	.29229E-03	7.731	7.484
105.729	104.000	269.38	259.31	27.88	.22309E-03	8.150	7.885
106.000	104.261	271.52	261.20	27.86	.21414E-03	8.215	7.948
107.797	106.000	285.77	273.68	27.74	.16440E-03	8.651	8.365
108.000	106.195	287.37	275.08	27.73	.15971E-03	8.700	8.412
109.866	108.000	302.15	287.89	27.60	.12323E-03	9.153	8.844
110.000	108.128	303.21	288.80	27.59	.12103E-03	9.185	8.875
111.937	110.000	318.54	301.93	27.45	.93788E-04	9.655	9.324
112.000	110.060	319.03	302.36	27.45	.93035E-04	9.670	9.338
114.000	111.991	334.85	315.76	27.31	.72443E-04	10.156	9.801
114.008	112.000	334.92	315.82	27.31	.72364E-04	10.158	9.804
116.000	113.921	350.66	329.00	27.18	.57072E-04	10.642	10.264
116.081	114.000	351.31	329.54	27.17	.56530E-04	10.662	10.283
118.000	115.849	366.46	342.08	27.04	.45444E-04	11.129	10.727
118.156	116.000	367.69	343.09	27.03	.44661E-04	11.167	10.763
120.000	117.776	382.24	355.00	26.90	.36539E-04	11.615	11.189

*Read X in heading as h_x.

TABLE 9*

MODEL F PRIME R

PROPERTIES OF TRANSITION ATMOSPHERE
MIDLAT (45N) JAN TO JACCHIA MODELS

45N JAN X = 94.79564 KM DEN X = .11856087E-02 G/M3
GEOPOT SCALE HT AT 117.7765 KM = 11.18876, AT X KM = 5.7474122
GRADIENT OF GEOP SCALE HT ABOVE X = .23677739E 00
GRADIENT OF MOL. TEMP. -2.1 FOR H BTWN 52. TO 85
GRADIENT OF MOL. TEMP. ZERO FOR H BTWN 85. AND X
GRADIENT OF MOL. TEMP. 8.08907 DEG/KM FOR H BTWN X AND 117.7765

ALTITUDE KM		TEMP DEG KELVIN		MOL WT	DENSITY	SCALE HT KM	
GEOMET	GEOPOT	MOL.	KINETIC		G/M3	GEOMET	GEOPOT
52.428	52.000	265.65	265.65	28.96	.65845E 00	7.905	7.776
54.000	53.545	262.41	262.41	28.96	.54579E 00	7.812	7.681
54.462	54.000	261.45	261.45	28.96	.51623E 00	7.785	7.653
56.000	55.510	258.28	258.28	28.96	.42843E 00	7.694	7.560
56.497	56.000	257.25	257.25	28.96	.40314E 00	7.664	7.530
58.000	57.475	254.15	254.15	28.96	.33504E 00	7.576	7.439
58.534	58.000	253.05	253.05	28.96	.31355E 00	7.544	7.407
60.000	59.438	250.03	250.03	28.96	.26100E 00	7.457	7.319
60.571	60.000	248.85	248.85	28.96	.24284E 00	7.424	7.284
62.000	61.401	245.91	245.91	28.96	.20251E 00	7.339	7.198
62.610	62.000	244.65	244.65	28.96	.18726E 00	7.303	7.161
64.000	63.362	241.79	241.79	28.96	.15648E 00	7.221	7.077
64.650	64.000	240.45	240.45	28.96	.14376E 00	7.182	7.038
66.000	65.321	237.67	237.67	28.96	.12040E 00	7.102	6.957
66.692	66.000	236.25	236.25	28.96	.10984E 00	7.061	6.915
68.000	67.280	233.56	233.56	28.96	.92235E-01	6.984	6.837
68.735	68.000	232.05	232.05	28.96	.83531E-01	6.940	6.792
70.000	69.237	229.45	229.45	28.96	.70334E-01	6.865	6.716
70.779	70.000	227.85	227.85	28.96	.63202E-01	6.819	6.669
72.000	71.193	225.34	225.34	28.96	.53380E-01	6.746	6.596
72.824	72.000	223.65	223.65	28.96	.47573E-01	6.697	6.547
74.000	73.148	221.24	221.24	28.96	.40315E-01	6.628	6.476
74.871	74.000	219.45	219.45	28.96	.35616E-01	6.576	6.424
76.000	75.102	217.14	217.14	28.96	.30294E-01	6.509	6.356
76.919	76.000	215.25	215.25	28.96	.26516E-01	6.454	6.301
78.000	77.054	213.04	213.04	28.96	.22643E-01	6.390	6.236
78.969	78.000	211.05	211.05	28.96	.19627E-01	6.332	6.178
80.000	79.005	208.94	208.94	28.96	.16833E-01	6.271	6.116
81.019	80.000	206.85	206.85	28.96	.14440E-01	6.210	6.055
82.000	80.955	204.84	204.84	28.96	.12442E-01	6.152	5.996
83.071	82.000	202.65	202.65	28.96	.10557E-01	6.088	5.932
84.000	82.904	200.75	200.75	28.96	.91433E-02	6.033	5.876
85.124	84.000	198.45	198.45	28.96	.76678E-02	5.966	5.809
86.000	84.851	196.66	196.66	28.96	.66776E-02	5.913	5.757
86.152	85.000	196.35	196.35	28.96	.65183E-02	5.904	5.747
87.179	86.000	196.35	196.35	28.96	.54773E-02	5.906	5.747
88.000	86.798	196.35	196.35	28.96	.47670E-02	5.908	5.747
89.235	88.000	196.35	196.35	28.96	.38676E-02	5.910	5.747
90.000	88.743	196.35	196.35	28.96	.33983E-02	5.911	5.747

*Read X in heading as h_x.

TABLE 9 CONCLUDED

MODEL F PRIME R CONTINUED

ALTITUDE KM GEOMET	ALTITUDE KM GEOPOT	TEMP DEG KELVIN MOL.	TEMP DEG KELVIN KINETIC	MOL WT	DENSITY G/M3	SCALE HT KM GEOMET	SCALE HT KM GEOPOT
91.292	90.000	196.35	195.75	28.88	.27309E-02	5.914	5.747
92.000	90.687	196.35	195.42	28.83	.24231E-02	5.915	5.747
93.351	92.000	196.35	194.79	28.73	.19283E-02	5.917	5.747
94.000	92.630	196.35	194.48	28.69	.17281E-02	5.919	5.747
95.410	94.000	196.35	193.83	28.59	.13616E-02	5.921	5.747
96.000	94.571	196.35	193.55	28.55	.12327E-02	5.922	5.747
96.230	94.795	196.35	193.44	28.54	.11856E-02	5.923	5.747
97.472	96.000	206.09	202.43	28.45	.92064E-03	6.219	6.033
98.000	96.512	210.23	206.24	28.41	.82976E-03	6.345	6.154
99.534	98.000	222.27	217.24	28.31	.62038E-03	6.711	6.506
100.000	98.451	225.92	220.55	28.28	.56979E-03	6.823	6.613
101.598	100.000	238.45	231.88	28.17	.42981E-03	7.205	6.980
102.000	100.389	241.60	234.71	28.14	.40136E-03	7.301	7.072
103.663	102.000	254.63	246.36	28.02	.30504E-03	7.698	7.453
104.000	102.325	257.26	248.71	28.00	.28907E-03	7.779	7.530
105.729	104.000	270.80	260.68	27.88	.22111E-03	8.193	7.927
106.000	104.261	272.92	262.54	27.86	.21231E-03	8.257	7.989
107.797	106.000	286.98	274.85	27.74	.16330E-03	8.688	8.400
108.000	106.195	288.57	276.23	27.73	.15868E-03	8.736	8.447
109.866	108.000	303.16	288.85	27.60	.12262E-03	9.183	8.874
110.000	108.128	304.20	289.75	27.59	.12045E-03	9.215	8.904
111.937	110.000	319.34	302.70	27.45	.93465E-04	9.680	9.347
112.000	110.060	319.83	303.11	27.45	.92718E-04	9.695	9.362
114.000	111.991	335.45	316.32	27.31	.72279E-04	10.174	9.819
114.008	112.000	335.52	316.38	27.31	.72201E-04	10.176	9.821
116.000	113.921	351.06	329.37	27.18	.56995E-04	10.654	10.276
116.081	114.000	351.70	329.90	27.17	.56456E-04	10.674	10.295
118.000	115.849	366.66	342.26	27.04	.45417E-04	11.135	10.732
118.156	116.000	367.87	343.27	27.03	.44637E-04	11.172	10.768
120.000	117.776	382.24	355.00	26.90	.36539E-04	11.615	11.189

TABLE 10*

MODEL GR

PROPERTIES OF TRANSITION ATMOSPHERE
SUBARCTIC (60N) JAN TO JACCHIA MODELS

60N JAN X = 96.23401 KM DEN X = .95286054E-03 G/M3
GEOPOT SCALE HT AT 117.9299 KM = 11.18876, AT X KM = 5.9113313
GRADIENT OF GEOP SCALE HT ABOVE X = .24324556E 00
GRADIENT OF MOL. TEMP. -2.6 FOR H BTWN 69. TO 86.
GRADIENT OF MOL. TEMP. ZERO FOR H BTWN 86. AND X
GRADIENT OF MOL. TEMP. 8.31004 DEG/KM FOR H BTWN X AND 117.9299

ALTITUDE KM		TEMP DEG KELVIN		MOL WT	DENSITY	SCALE HT KM	
GEOMET	GEOPOT	MOL.	KINETIC		G/M3	GEOMET	GEOPOT
69.666	69.000	246.15	246.15	28.96	.59480E-01	.000	7.205
70.000	69.326	245.30	245.30	28.96	.57037E-01	7.330	7.180
70.687	70.000	243.55	243.55	28.96	.52287E-01	7.279	7.129
72.000	71.285	240.21	240.21	28.96	.44213E-01	7.182	7.031
72.730	72.000	238.35	238.35	28.96	.40236E-01	7.128	6.977
74.000	73.242	235.12	235.12	28.96	.34092E-01	7.034	6.882
74.774	74.000	233.15	233.15	28.96	.30783E-01	6.977	6.825
76.000	75.198	230.03	230.03	28.96	.26142E-01	6.886	6.733
76.819	76.000	227.95	227.95	28.96	.23410E-01	6.826	6.672
78.000	77.153	224.95	224.95	28.96	.19931E-01	6.738	6.585
78.865	78.000	222.75	222.75	28.96	.17690E-01	6.674	6.520
80.000	79.107	219.87	219.87	28.96	.15105E-01	6.590	6.436
80.913	80.000	217.55	217.55	28.96	.13280E-01	6.523	6.368
82.000	81.060	214.79	214.79	28.96	.11375E-01	6.442	6.287
82.963	82.000	212.35	212.35	28.96	.99004E-02	6.371	6.216
84.000	83.011	209.72	209.72	28.96	.85103E-02	6.294	6.139
85.013	84.000	207.15	207.15	28.96	.73272E-02	6.219	6.064
86.000	84.961	204.65	204.65	28.96	.63228E-02	6.145	5.990
87.065	86.000	201.95	201.95	28.96	.53815E-02	6.066	5.911
88.000	86.910	201.95	201.95	28.96	.46131E-02	6.068	5.911
89.118	88.000	201.95	201.95	28.96	.38367E-02	6.070	5.911
90.000	88.858	201.95	201.95	28.96	.33182E-02	6.072	5.911
91.172	90.000	201.95	201.39	28.88	.27354E-02	6.074	5.911
92.000	90.804	201.95	200.99	28.83	.23872E-02	6.076	5.911
93.228	92.000	201.95	200.40	28.74	.19502E-02	6.078	5.911
94.000	92.750	201.95	200.03	28.69	.17178E-02	6.079	5.911
95.285	94.000	201.95	199.41	28.60	.13904E-02	6.082	5.911
96.000	94.694	201.95	199.07	28.55	.12363E-02	6.083	5.911
97.344	96.000	201.95	198.43	28.46	.99133E-03	6.086	5.911
97.584	96.234	201.95	198.31	28.44	.95286E-03	6.086	5.911
98.000	96.637	205.30	201.40	28.41	.87601E-03	6.188	6.009
99.403	98.000	216.63	211.79	28.32	.66576E-03	6.532	6.341
100.000	98.578	221.44	216.17	28.28	.59507E-03	6.678	6.482

*Read X in heading as h_x.

TABLE 10 CONCLUDED

MODEL GR CONTINUED

ALTITUDE KM GEOMET	GEOPOT	TEMP DEG KELVIN MOL.	KINETIC	MOL WT	DENSITY G/M3	SCALE HT KM GEOMET	GEOPOT
101.464	100.000	233.25	226.89	28.18	.45628E-03	7.038	6.827
102.000	100.519	237.56	230.79	28.14	.41547E-03	7.169	6.954
103.526	102.000	249.87	241.84	28.03	.32095E-03	7.544	7.314
104.000	102.458	253.68	245.24	28.00	.29706E-03	7.660	7.425
105.590	104.000	266.49	256.62	27.89	.23094E-03	8.051	7.800
106.000	104.396	269.78	259.53	27.86	.21687E-03	8.152	7.897
107.655	106.000	283.11	271.23	27.75	.16951E-03	8.559	8.287
108.000	106.333	285.88	273.65	27.73	.16128E-03	8.643	8.368
109.721	108.000	299.73	285.68	27.61	.12664E-03	9.067	8.773
110.000	108.269	301.96	287.62	27.59	.12191E-03	9.135	8.839
111.789	110.000	316.35	299.97	27.47	.96114E-04	9.576	9.260
112.000	110.203	318.04	301.42	27.45	.93527E-04	9.628	9.309
113.858	112.000	332.97	314.09	27.32	.73982E-04	10.085	9.746
114.000	112.137	334.11	315.05	27.31	.72702E-04	10.120	9.780
115.928	114.000	349.59	328.05	27.18	.57677E-04	10.595	10.233
116.000	114.069	350.16	328.53	27.18	.57195E-04	10.613	10.250
117.999	116.000	366.21	341.85	27.04	.45489E-04	11.106	10.719
118.000	116.000	366.21	341.85	27.04	.45488E-04	11.106	10.719
120.000	117.929	382.24	355.00	26.90	.36540E-04	11.600	11.189

correspond respectively to the h_x altitudes of models Er, Fr, and Gr where the temperature altitude gradients of these models increase abruptly from 0 to some positive value. Similarly, the sharp brakes at altitudes 99.522 to 103.87, correspond to the h_x altitudes of models Ar to Dr, respectively.

Application of the Exact Method to More Complicated Models by Inference

These simple models of first-degree complexity whose temperature-altitude profiles are depicted in Figures 1 and 3 are not the only models which can be developed to fit the prescribed boundary conditions; an infinite number of more complex models are possible. These simple models based upon a single application of Equation (4) are useful, however, in establishing limiting conditions for models of second-degree complexity which are characterized by the following conditions:

1. The altitude boundaries h_b and h_a, for which altitudes the corresponding values of T_M and ρ are given, encompass n altitude layers with interfaces at h_x, h_2, h_3,..., and h_{a-1}.

2. The lowest layer extending upward from h_b to h_x is an isothermal layer.

3. The successive layers h_x to h_2, h_2 to h_3,... h_{a-1} to h_a are characterized by monotonically increasing values of temperature-altitude gradient.

Models of the first-degree complexity from Equation (4) bear the following relationships to models of the second-degree complexity assuming identical boundary conditions at h_b and h_a for both types of models:

1. The value of h_x from Equation (4) represents the greatest possible value of h_x for models of second-degree complexity.

2. The temperature-altitude gradient expressed by Equation (5) for models of the first-degree complexity represents the smallest possible gradient which the upper-most layer of models of the second-degree complexity may have.

The models of the first-degree complexity presented in Figures 1 through 4 and in the related tables plus this relationship to models of the second-degree complexity served as a guide to Task Group IX of COESA in the development of cut-and-try transition models of the second-degree complexity between the Cole-Kantor models below 90 km and certain Jacchia models above 120 km.

REVISIONS TO THE UNITED STATES STANDARD ATMOSPHERE

Reason for Revision

The presently available atmospheric density data suggest that the density of the mean or 45-degree atmosphere is significantly greater at 90 km than that of the United States Standard Atmosphere. Consequently, the COESA task group working on the problems of transition atmospheres recommended that the density of its mean atmosphere be increased by 10 percent at 90 km (later changed to 14 percent). This mean atmosphere designated as a Spring-Fall model would retain the standard-atmosphere temperature and density to as great an altitude as possible below 90 km, and would match the Jacchia model values [7] at 120 km.

General Approach

From considerations involving the hydrostatic equation and the equation of state, it may be shown that the variations of the density ρ at any altitude h_b relative to a fixed value of density ρ_b at altitude h_b is determined by the variation of the reciprocal of $\bar{T}_M$, the mean value of the molecular scale temperature for the interval h_b to h; i.e.,

$$\ell n\ \rho = \ell n\ \rho_b - \frac{Q(h - h_b)}{\bar{T}_M} \qquad (6)$$

Consequently, for a fixed value of density and temperature at altitude h_b, the density at altitude h may be increased only if the mean temperature between h_b and h is increased. This is accomplished by shifting the temperature-altitude gradient above altitude h_b to more positive values.

As a first consideration one could lower the onset of the isothermal layer from 79 km' to 76 km' and thereby increase the mesopause isothermal temperature from 180.65 to 192.65°K, and simultaneously increase the 90-km' density by about 10 percent. Such a lowering of the onset of the mesopause was unacceptable to some members of the task group because of the existing association of 79 km' with this mesopause point.

Another possibility is to change the slope of some or all of that linear segment of temperature-altitude profile immediately below the mesopause isothermal layer (between 61 km' and 79 km') from the existing value of $-4^{\circ}K/km'$ to some more positive value, and thereby increase the temperature at altitudes near 79 km'. Such a process could increase the temperature of the mesopause isothermal layer without substantially altering the altitude of the base of the mesosphere layer. In order that relatively round numbers may be retained for the temperature-altitude gradients and temperatures, the selection of the gradients for the investigation was limited to a set of values having successive increments of $+0.1^{\circ}K/km'$, from $-4.0^{\circ}K/km'$ to more positive values.

Detailed Computation Considerations

The program for digital machine operation involved the following procedures and equations.

(1) Beginning with the standard-atmosphere values of T_M and ρ at h_b, i.e., $(T_M)_b$ and ρ_b respectively, compute $(T_M)_i$ and ρ_i, the values of T_M and ρ respectively at h_i the base of the isothermal layer by means of the expressions:

$$(T_M)_i = (T_M)_b + L_b'(h_i - h_b), \tag{7}$$

and

$$\rho_i = \rho_b \left[\frac{(T_M)_b}{(T_M)_i}\right]^{(1 + Q/L_b')} \tag{8}$$

where

$$Q = \frac{GM_o}{R} = 34.1631947^{\circ}K/km, \tag{9}$$

and

L_b = successively each of the N members of the set -4.0, -3.9, -3.8, ..., while

h_i = successively each of the 15 members of the set 76, 77, 78, ... 90 km'

The use of these N values of L_b' and 15 values of h_i leads to 15N sets of values of $(T_M)_i$ and ρ_i for each base altitude selected.

(2) Using the 15N sets of values of $(T_M)_i$ and ρ_i obtained from Equations (7) and (8) and their related boundary conditions, compute $(T_M)_{90}$ and ρ_{90} the temperature and density, respectively, at 90 km' by means of the expressions:

$$(T_M)_{90} = (T_M)_i \tag{10}$$

and

$$\rho_{90} = \rho_i \exp\left(- \frac{(90 - h_i)Q}{(T_M)_i}\right) \tag{11}$$

Since there are 15N sets of values of h_i, ρ_i, and $(T_M)_i$, there are 15N sets of values of h_{90} and $(T_M)_{90}$, for each value of h_b employed. The range of these 15N values of ρ_b is large, and only a selected few are found to be close to the desired 10 or 14 percent increase over the standard-atmosphere value of ρ_{90}.

(3) These selected values of ρ_{90} and their related values of $(T_M)_{90}$ are introduced into Equation (4) as ρ_b and $(T_M)_b$ for h_b= 90 km', when h_a is taken to be 117.7765 km', the geopotential equivalent of 120 geometric kilometers for 45° latitude, while ρ_a and $(T_M)_a$ are taken to be the Jacchia-Model values of ρ and T_M respectively at that altitude. A solution of Equation (4) yielding a value h_x between 90 km' and 117.7765 km' indicates that the particular values of ρ_{90} and $(T_M)_{90}$ are realistic with respect to the 120-km boundary condition imposed by the Jacchia Model. Not all of the values selected from step (2) survive this test.

The calculations were performed for each of two values of h_b, 61 km', and 69 km'. The first of these was chosen on the basis that 61 km' is the altitude of the base of that layer immediately below the isothermal layer. The second of these altitudes was arbitrarily chosen to increase the altitude above which the standard atmosphere would be modified, while simultaneously allowing enough of an interval between h_b and 90 km to bring about the desired increase in the 90-km' density. The values of $(T_M)_b$ and ρ_b for each of the two cases are as follows:

h_b	$(T_M)_b$	ρ_b
61 km'	252.65°K	.25 104 x 10^{-3}kg m^{-3}
69 km'	220.65°K	.90 430 x 10^{-4}kg m^{-3}

Results of Calculations for h_b = 61 km'

For the case h_b = 61 km', nine values of L_b' were employed, -4.0, -3.9, ... -3.3, -3.2°K/km' resulting in 135 values of ρ_{90}. These 135 values are plotted as a function of h_i in Figure 5 where each set of values of ρ_{90} associated with any particular value of L_b' were connected by a single smooth curve. Thus, there are nine curves labeled J through R, one for each of the nine values of L_b'.

Two horizontal lines represent densities of 1.1 times and 1.14 times the standard-atmosphere values of density, respectively. Small circles on each of the curves (J) through (P) near the line for $1.1\rho_{90,std}$ represent those values of ρ_{90} which are approximately 10 percent greater than the corresponding standard-atmosphere values of density, and which simultaneously are associated with an altitude h_i having a value equal to an integral multiple of 1 km and with a particular gradient L_b' (between 61 km' and h_i) having a value expressed in an integral multiple of 0.1°K/km. Note that there are two circles on curve O as well as on curve N.

Temperature-altitude profiles corresponding to eight of the nine small circles near the line representing $1.1\rho_{90,std}$ are shown in Figure 6. The model designated O' corresponds to the circle to the right of the minimum of curve O on Figure 5. Altitude h_x, the upper end of the isothermal layer of each model shown, corresponds to the greatest altitude of the isothermal layer consistent with the condition of connecting to the 120-km values of T_M and ρ for the Jacchia Models as determined by Equation (4). Figure 6 contains no model N' consistent with the circle to the right of the minimum of curve N on Figure 5 because such a model can in no way be made consistent with the Jacchia-Model value of T_M and ρ at 120 km.

The line of Figure 5 corresponding to 1.14 times $\rho_{90,std}$ also has a set of related circles on the curves L through Q. (No circle appears on curve R, since ρ_{90} on any R-type model would be at least 1.16 times $\rho_{90,std}$.) Curves Q and P both have two circles corresponding to densities near 1.14 times $\rho_{90,std}$. Figure 7 shows temperature-altitude profiles for models consistent with the circles near the line for 1.14 x $\rho_{90,std}$. These models depart from the standard atmosphere above 61 km', have densities 14 percent greater than the standard-atmosphere at 90 km, and can again be joined to the Jacchia-Model values of T_M and ρ at 120 km. The salient features of sixteen models departing from the standard atmosphere at 61 km and matching the Jacchia models at 120 km are given in Table 11.

Results of Calculations for h_b = 69 km'

For the case h_b = 69 km', 11 values of L_b' were employed, -4.0, -3.9 ... -3.1, -3.0°K/km' resulting in 165 values of ρ_{90}. These 165 values were plotted as a function of h_i with a single smooth curve connecting each set of values of ρ_{90} associated with a single value of L_b'. An abbreviated

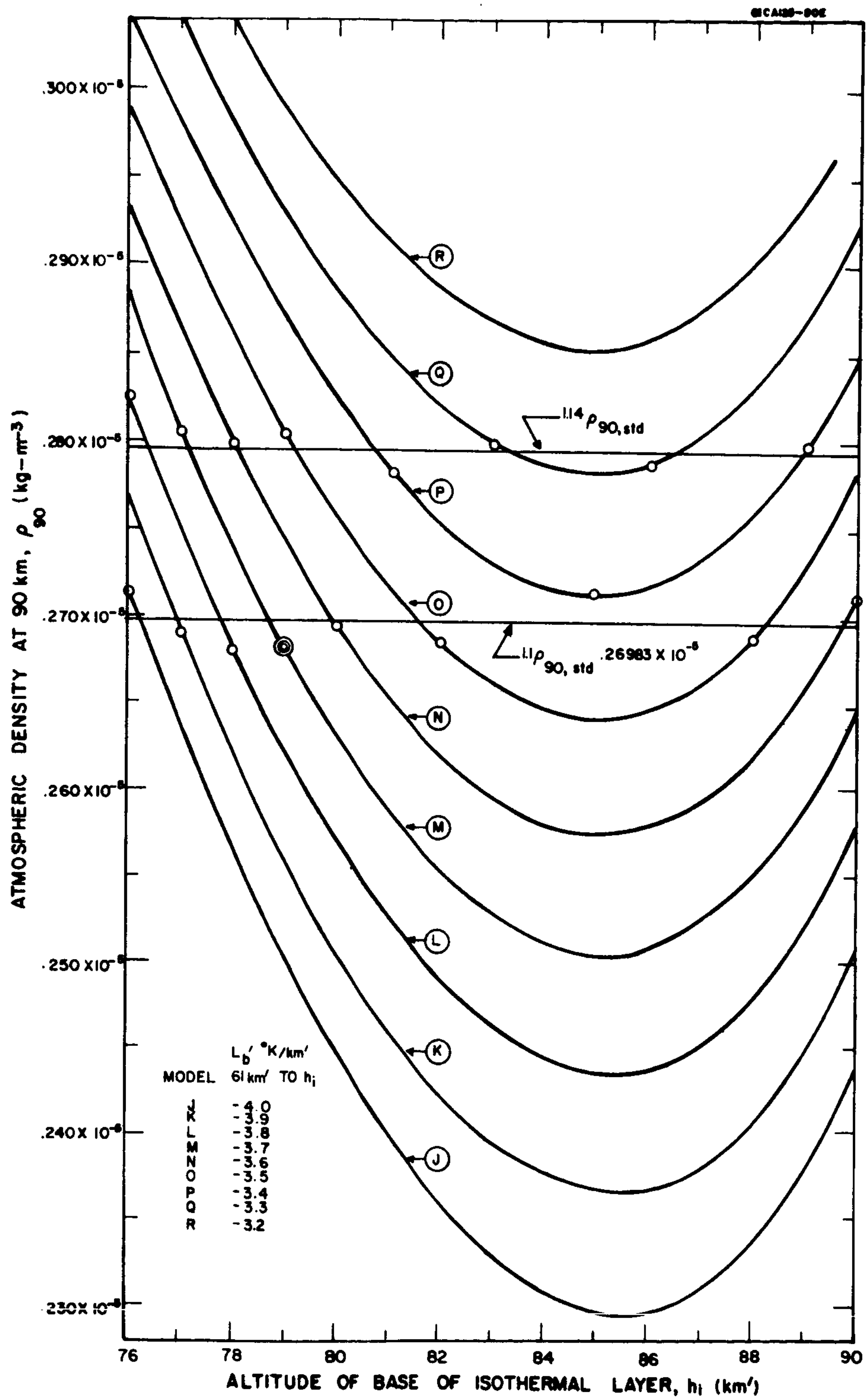

Figure 5. Values of atmospheric density at 90 km′ as a function of the altitude of the base of an isothermal layer h_i and the corresponding values of $(T_M)_i$ and ρ_i deduced from the standard atmosphere values of T_M and ρ at 61 km′ and particular values of negative temperature gradients between 61 km and h_i corresponding to models J through R.

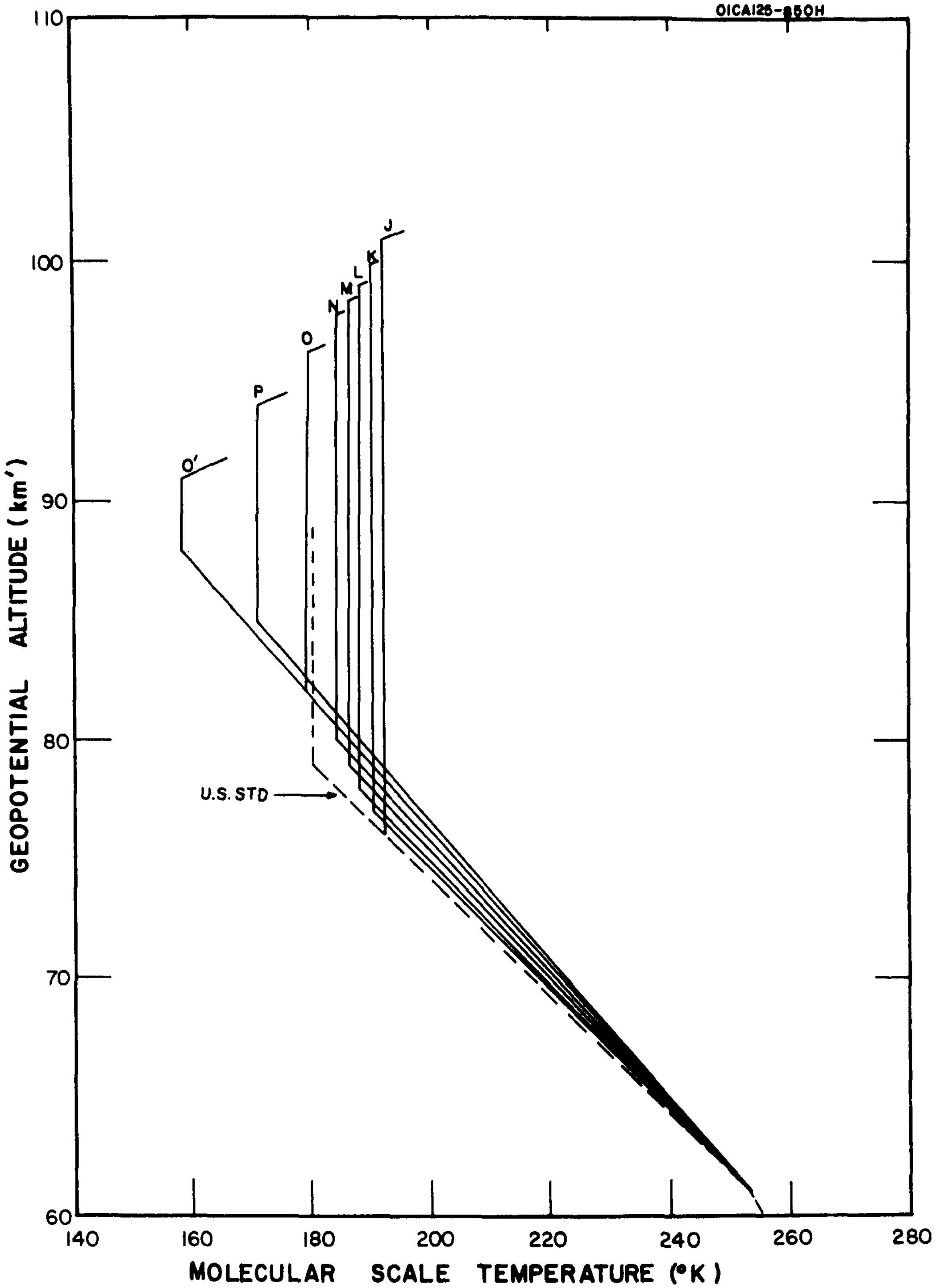

Figure 6. Temperature-altitude profiles of possible models which depart from the U.S. Standard Atmosphere at 61 km′, yield densities at 90 km′ of about 1.1 times the corresponding standard-atmosphere density and yet are capable of yielding the Jacchia-Model densities at 120 km.

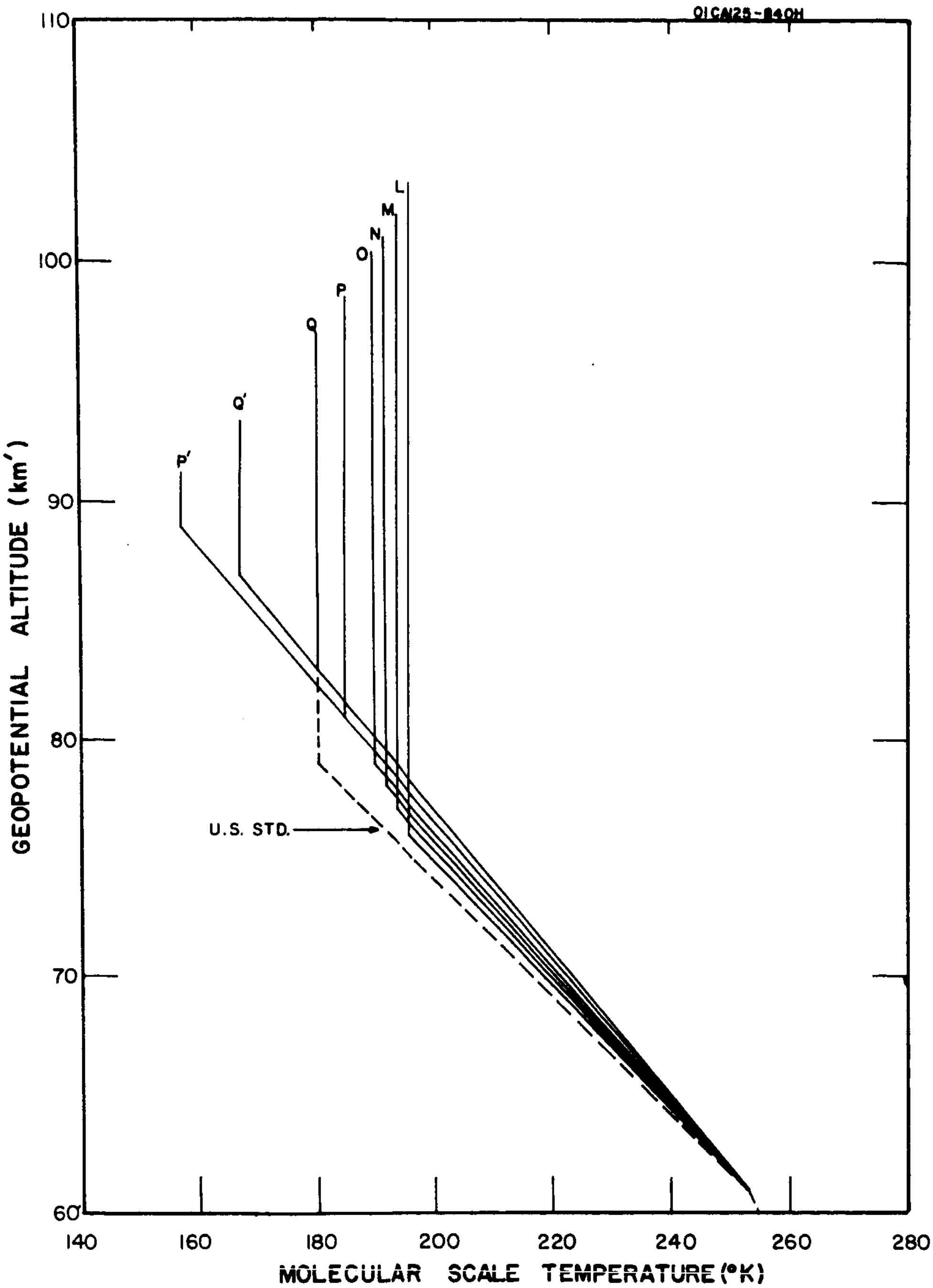

Figure 7. Temperature-altitude profiles of possible models which depart from the U.S. Standard Atmosphere at 61 km′ yields densities at 90 km′ of about 1.14 times the corresponding standard atmosphere densities, and yet are capable of yielding the Jacchia-Model densities at 120 km.

TABLE 11

All possible cases for increasing the standard-atmosphere values of atmospheric density ρ_{90} by approximately 10 percent and 14 percent consistent with the following boundary conditions:

1. Standard-atmosphere values of T_M and ρ remain unchanged at altitudes of 61 km′ and below.
2. Jacchia Model values of T_M and ρ are matched at geometric altitudes of 120 km.
3. A single temperature-altitude gradient, expressible as an integral multiple of 0.1°/km′, exists between 61 km′ and h_i the base of the mesopause level.
4. The mesopause isothermal layer begins at an altitude represented by an integral multiple of 1 km′.

	~10% Increase in ρ_{90}						~14% Increase in ρ_{90}					
L'_b 61 km′ to h_i °K/km′	Model	h_i km′	$(T_M)_i$ °K	ρ_{90} k_g m^{-3}	X km′	$L'_{b,min}$ °K/km′	Model	h_i km′	$(T_M)_i$ °K	ρ_{90} k_g m^{-3}	X km′	$L'_{b,min}$ °K/km′
-4.0	10J61	76	192.65	0.27145E-05	100.93394	11.25683						
-3.9	10K61	77	190.25	0.26918E-05	99.90099	10.74060						
-3.8	10L61	78	188.05	0.26811E-05	99.05842	10.37466	14L61	76	195.65	0.28243E-05	102.83149	12.48536
-3.7	10M61	79	186.05	0.26822E-05	98.38964	10.11994	14M61	77	193.45	0.28062E-05	101.84889	11.85324
-3.6	10N61	80	180.25	0.26947E-05	97.87234	9.94736	14N61	78	191.45	0.27999E-05	101.05141	11.40764
-3.5	10Ø61	82	179.15	0.26869E-05	96.23549	9.42824	14Ø61	79	189.65	0.28051E-05	100.42446	11.09920
-3.4	10P61	85	171.05	0.27127E-05	94.09925	8.91969	14P61	81	184.65	0.27835E-05	98.55297	10.27874
-3.3							14Q61	83	180.05	0.27990E-05	97.16112	9.80791
-3.3							14Q′61	86	166.85	0.28041E-05	93.47573	8.86366
-3.4							14P′61	89	157.45	0.28012E-05	91.26027	8.47759
-3.5	10Ø′61	88	158.15	0.269023	90.91645	8.34301						

(A review of the computer program with which X and $L_{M,min}$ were determined indicates that these values are consistent with the Jacchia models at 120 km rather than with the United States Standard Atmosphere. The differences between these two models are small; however, at 120 km values of X and $L_{M,min}$ would not be much different if the standard-atmosphere values had been used).

version of this eleven-curve graph is presented as Figure 8. Again, horizontal lines representing values of ρ_{90} equal to 1.10 $\rho_{90,std}$ and to 1.14 $\rho_{90,std}$ are presented. Those values of ρ_{90} simultaneously consistent with values of h_i equal to integral multiplies of 1 km' and 1.10 $\rho_{90,std}$ are designated with small circles. Models R and T both have two such circled values. Models K, L, N, O, and Q have no such circled points since, for h_i equal to an integral multiple of 1 km', the related value of ρ_{90} would be more removed from 1.10 $\rho_{90,std}$ than those values of ρ_{90} for one or more other models.

Figure 9 shows the temperature-altitude profiles consistent with each of these circled density values near 1.10 $\rho_{90,std}$ except for the right-hand circle of model R. This value of density and its related temperature were found, by Equation (4), to be incompatible with the required 120-km boundary conditions. The tops of the isothermal layers for the other models each show the greatest possible altitude of the isothermal layer consistent with the 120-km boundary conditions.

Figure 8 also shows a set of small circles associated with a density value of 1.14 $\rho_{90,std}$. None of the curves has second values of density near to the 1.14 $\rho_{90\ std}$ value, and only 3 of the curves contain density-altitude points which simultaneously satisfy the boundary condition relative to h_i and the condition of having a value near to 1.14 $\rho_{90,std}$. The three related temperature-altitude profiles are shown in Figure 10, and again the top of these isothermals represent the greatest possible altitude of that isothermal consistent with the 120-km boundary condition.

The salient features of the 10 possible models departing from the standard atmosphere at 69 km and meeting the specified boundary conditions of the Jacchia model are summarized in Table 12. It is interesting to note from Tables 11 and 12 as well as from Figures 6, 7, 9 and 10, that as the value of h_b is increased from 61 to 69 km' the number of possible models, consistent with any fixed set of the remaining boundary conditions, decreases. This is particularly true for ρ_{90} selected to be near 1.14 $\rho_{90,std}$, where the number of possible models is seen to drop from eight to three.

From working copies of the supplementary atmospheres document [13] (1966), it appears that the model finally selected by Cole and Kantor without any specific discussion in the working-group meeting is one consistent with model T of Figure 8 with h_i taken to be 79 km'. This yields a density which is only about 13 percent greater than the standard-atmosphere density at 90 km, but has the advantage of retaining the 79 km' value as the base of the isothermal layer as in the existing U.S. Standard Atmosphere. The advantage of conformity in the matter of the base of the isothermal layer was considered to be more important than the reality of the density value. The corresponding temperature-altitude profile for this model is shown as a dash-dot line in Figure 10.

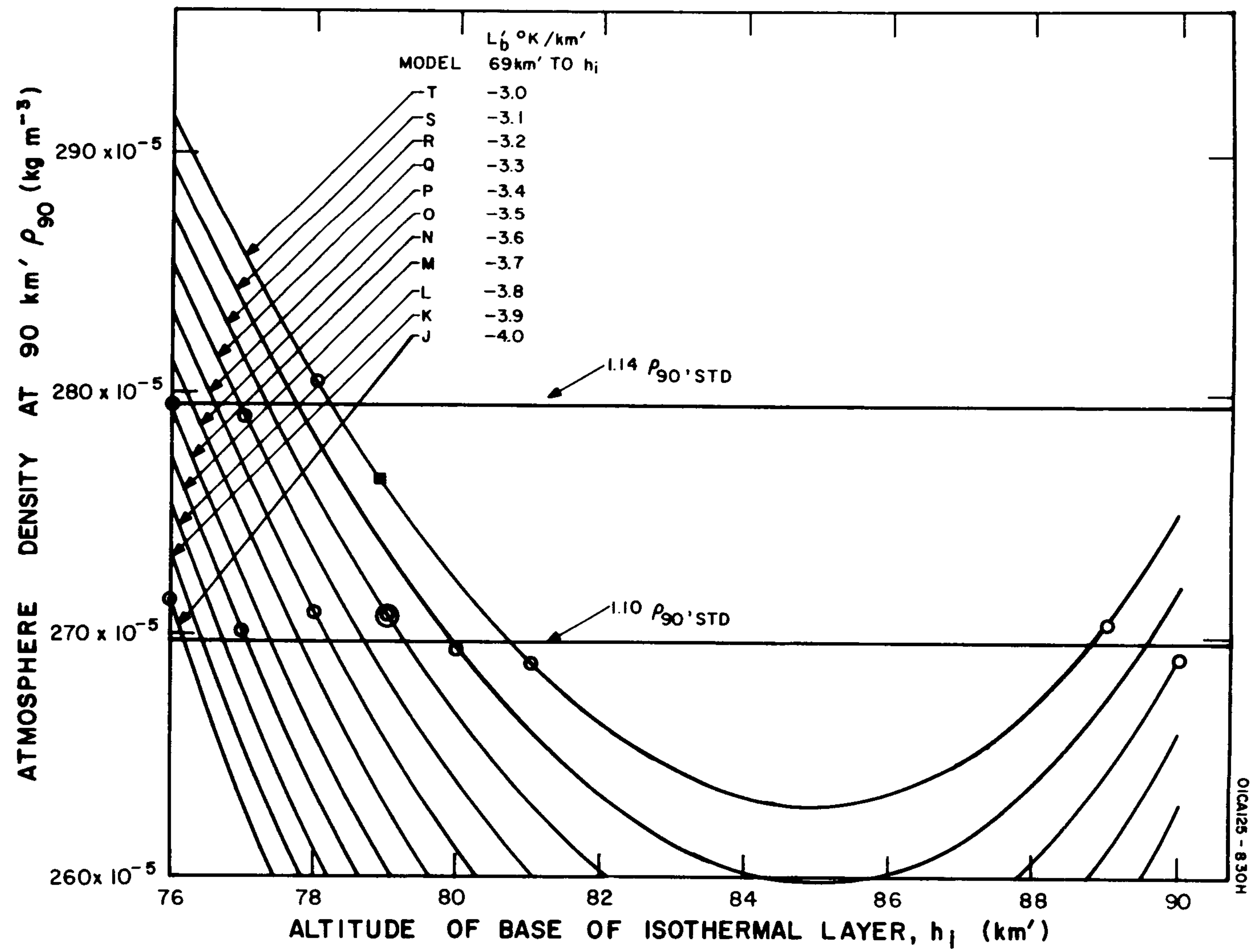

Figure 8. Values of atmospheric density at 90 km′ as a function of the altitude of the base of an isothermal layer h_i and the corresponding values of $(T_M)_i$ and p_i deduced from the standard atmosphere values of T_M and ρ at 69 km′ and particular values of negative temperature gradients gradients between 69 km′ and h_i corresponding to models J through R.

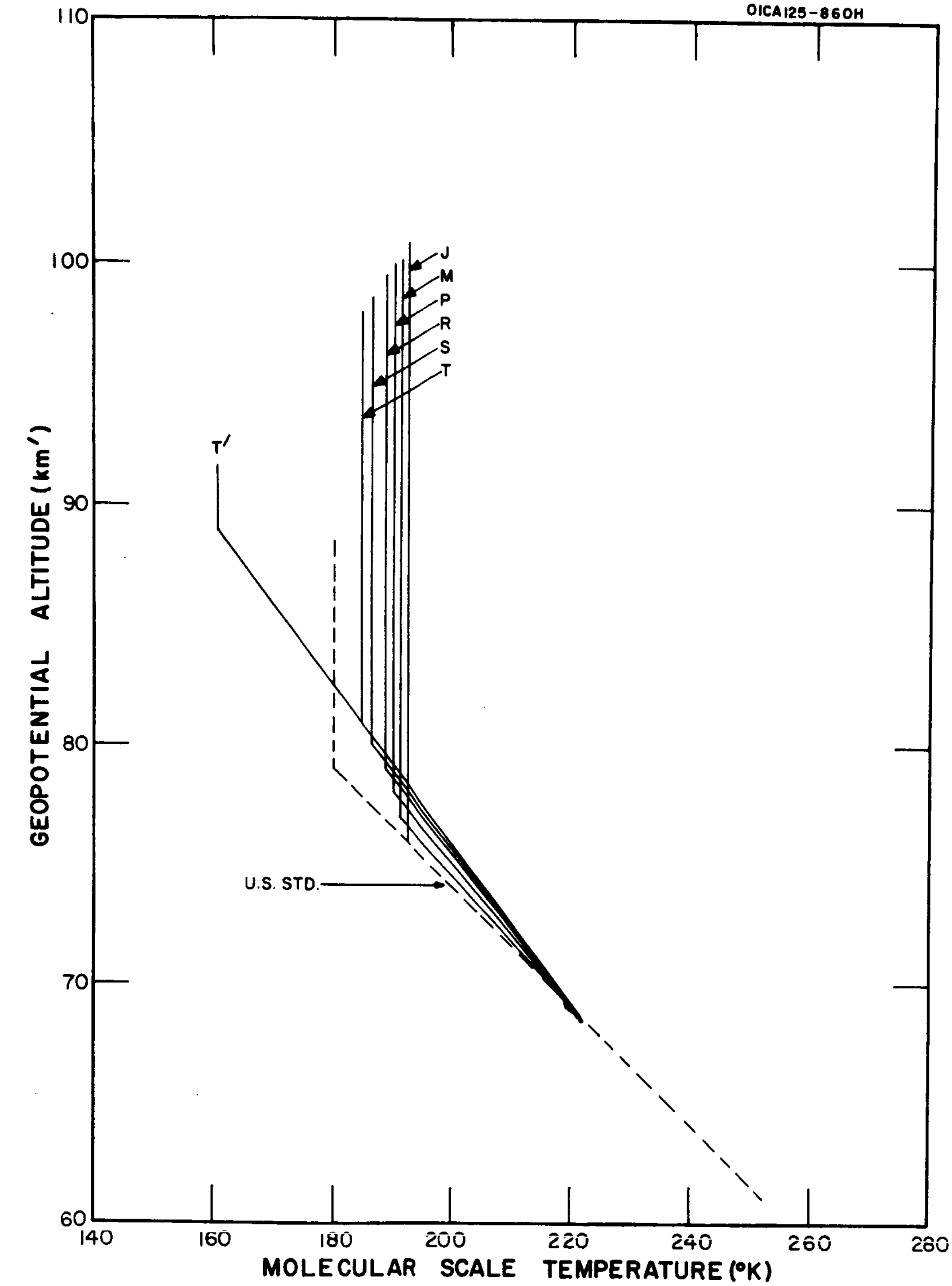

Figure 9. Temperature-altitude profiles of possible models which depart from the U.S. Standard Atmosphere at 69 km′ yield densities at 90 km′ of about 1.1 times the corresponding standard-atmosphere densities and yet are capable of yielding the Jacchia-Model densities at 120 km.

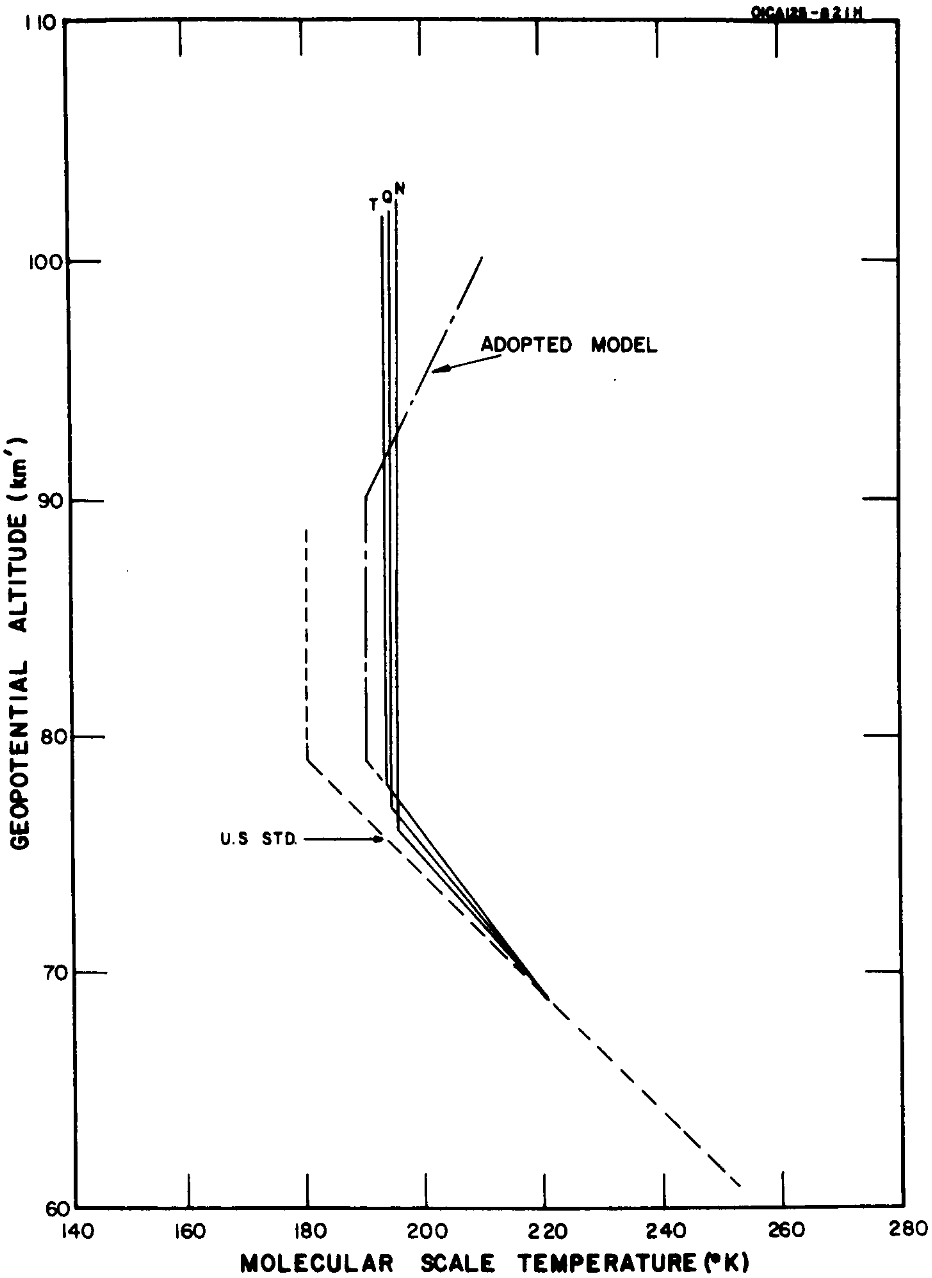

Figure 10. Temperature-altitude profiles of possible models which depart from the U.S. Standard Atmosphere at 69 km′ yield densities at 90 km′ of about 1.14 times the corresponding standard-atmosphere densities and yet are capable of yielding the Jacchia-Model densities at 120 km.

TABLE 12

Salient features of the temperature-altitude profile of all possible models for increasing $\rho_{90,std}$, the standard-atmosphere value of atmospheric density at 90 km by approximately 10 percent and 14 percent respectively consistent with the following boundary conditions:

1. Standard-atmosphere values of T_M and ρ remain unchanged at altitudes of 69 km' and below.
2. Jacchia-Model values of T_M and ρ are matched at geometric altitudes of 120 km.
3. A single temperature-altitude gradient expressible as an integral multiple of 0.1°K/km' exists between 69 km' and h_i, the base of the mesopause isothermal layer.
4. The mesopause isothermal layer begins at an altitude represented by an integral multiple of 1 km'.

	~ 10 Percent Increase in ρ_{90}						~ 14 Percent Increase in ρ_{90}					
L'_b °K/km	Model	h_i km'	$(T_M)_i$ °K	ρ_{90} kg/m³	X km'	$L'_{b,min}$ °K/km'	Model	h_i km'	$(T_M)_i$ °K	ρ_{90} kg/m³	X km'	$L'_{b,min}$ °K/km'
-4.0	10J69	76	192.65	.27145E-05	100.29	11.25	--	--	--	---	--	--
-3.7	10M69	77	191.05	.27020E-05	100.25	10.91	--	--	--	---	--	--
-3.6	10N69	--	--	--	--	--	14N69	76	195.45	$.27952 \times 10^{-5}$	102.55	12.27
-3.4	10P69	78	190.05	.27093E-05	99.94	10.78	--	--	--	---	--	--
-3.3	10Q69	--	--	--	--	--	14Q69	77	194.25	$.27911 \times 10^{-5}$	102.05	11.96
-3.2	10R69	79	188.65	.27087E-05	99.44	10.56	--	--	--	---	--	--
-3.1	10S69	80	186.55	.26941E-05	98.63	10.22	--	--	--	---	--	--
-3.0	10T69	81	184.65	.26891E-05	97.97	9.97	14T69	78	193.65	$.28059 \times 10^{-5}$	101.92	11.89
-3.0	10T'69	89	160.65	.27057E-05	91.53	8.44	--	--	--	---	--	--

(A review of the computer program with which X and $L_{M,min}$ were determined indicates that these values are consistent with the Jacchia models at 120 km rather than with the U.S. Standard Atmosphere. The differences between these two models are small at 120 km; consequently, values of X and $L_{M,min}$ would not be much different if the standard-atmosphere values had been used).

EXTENSION OF THE METRIC TABLES OF THE U. S. STANDARD ATMOSPHERE BY INTERPOLATION

Task Group IX of COESA has designated that the Supplementary Atmospheres [13] are to contain graphs (and possibly tables) in which the several season-latitude models are compared with the U. S. Standard Atmosphere. This comparison is to be in the form of percentage deviation of the densities of each of the supplementary model atmospheres from that of the Standard Atmosphere as a function of altitude from 0 to 120 km. In this altitude region the supplementary atmospheres are defined by a function of molecular scale temperature expressed as a series of segments, each linear in geopotential. The principal tables of the various atmospheric properties are presented as a function of integral multiples of 1 geopotential kilometer or small fractions thereof, and the comparisons are to be made at these values of geopotential.

Between altitudes of 0 and 90 geopotential kilometers (km′) the comparisons are easily made, but considerable problems develop when one attempts to make the comparisons in the altitude region of 90 to 120 km′. In this region, the U. S. Standard Atmosphere is defined in terms of a linearly segmented temperature-altitude profile whose segments are linear relative to geometric altitude, and the various atmospheric properties are tabulated only as a function of integral multiples of 1 geometric kilometer. It has become necessary therefore to extend the geopotential-dependent tabulations of the Standard Atmosphere to include the geometric altitude region 90 to 120 km.

The most reasonable approach would seem to be to use the same equations employed in the calculation of the Standard Atmosphere for that altitude region; i.e., the equations for computing $P(Z)$ the pressure at altitude Z or $\rho(Z)$ the density at altitude Z. Instead of an evaluation of these equations for integral multiples of 1 geometric kilometer, as for the existing atmospheric tables, these equations would be evaluated for those geometric altitudes equivalent to integral multiples of 1 geopotential kilometer, as defined by the same expressions relating Z to H in the Standard Atmosphere.

Unfortunately the Standard-Atmosphere document is lacking in at least the following important details necessary to set up the program for the above suggested calculation:

1. A functional expression for directly relating H to Z or vice versa.

2. A functional expression for directly computing pressure or density as a function of either geopotential or geometric altitude for the case when the defining temperature-altitude function is linear in geometric altitude.

3. A highly accurate value of the pressure and density at 90 km suitable to be used as a reference-level value for the calculation of pressure or density above 90 km.

4. A combined value of the constant group $(GM_o)/R$ as used in the calculation of pressure and density at the various critical altitudes 11, 20, 32, 47, 53, 61, and 79 geopotential kilometers as well as for that value of H equal to 90 geometric kilometers.

Because of the absence of the above-listed critical equations and constants and a desire to avoid the effort required to determine these needs, the rigorously correct calculation was at first bypassed in favor of an approximate method involving a simple interpolation.

Development of Interpolation Method

When the problem of generating standard-atmosphere tables as a function of geopotential was first presented, it appeared that it could be readily solved by an approximation method involving interpolation between successive geometric-altitude-related entries of the standard-atmosphere tables to the desired value, for a non-integral value of Z corresponding to H_i the integral-geopotential-kilometer value of H by means of some simple relationship. One could, for example, determine $T(H_i)$ the temperature at geopotential H_i, which temperature would be identical to $T(Z)$ the temperature at the related altitude Z, by an interpolation between $T(Z_i)$ and $T(Z_{i+1})$ where these quantities represent values of T corresponding to two successive integral multiples, i and i+1, of 1 geometric kilometer such that $Z_i < Z < Z_{i+1}$. Similarly, interpolation would be performed between $T_M(Z_i)$ and $T_M(Z_{i+1})$, between $P(Z_i)$ and $P(Z_{i+1})$ as well as between $\rho(Z_i)$ and $\rho(Z_{i+1})$ where these pairs of quantities represent respectively the values of T_M, P, and ρ at geometric altitudes Z_i and Z_{i+1}. These interpolations would yield values of $T(H_i)$, $T_M(H_i)$, $P(H_i)$ and $\rho(H_i)$ which are respectively the values of T, T_M, P, and ρ corresponding to those values of Z related to H_i by the following relationship [1, 14]

$$Z = \frac{rH_{i-n}}{\dfrac{g_o r}{G} - H_{i-n}} \tag{12}$$

where

$$r = 6,356,766 \text{ m}$$

$$g_o = 9.80665 \text{ m sec}^{-2}$$

$$G = 9.80665 \text{ m}^2 \text{ sec}^{-2} (\text{m}')^{-1} \text{ such that}$$

$$g_o r/G = 6,356,766 \text{ m}'$$

and where H_{i-n} is geopotential expressed in integral multiples (i-n) of 1 km′.

Thus, using a linear interpolation for T and T_M we have

$$T(H_i) = T(Z_i) + \left[\frac{T(Z_{i+1}) - T(Z_i)}{1 \text{ km}}\right] \Delta Z \tag{13}$$

where

$$\Delta Z = \left[\frac{r H_{i-n}}{\frac{g_o r}{G} - H_{i-n}} - Z_i\right] \tag{14}$$

and

$$Z_i < \left(\frac{r H_{i-n}}{\frac{g_o r}{G} - H_{i-n}}\right) < Z_{i+1}$$

requiring n to take on values of 1 or 2 as necessary for $90 \leq i \leq 120$, so that $\Delta Z < 1$ km.

Similarly,

$$\begin{aligned} T_M(H_i) &= T_M(Z_i) + \left[\frac{T_M(Z_{i+1}) - T_M(Z_i)}{1 \text{ km}}\right] \Delta Z \\ &= T_M(Z_i) + L \,\Delta Z \end{aligned} \tag{15}$$

where L is the temperature-altitude gradient as defined by the Standard Atmosphere for the appropriate altitude region, and where ΔZ is as defined in Equation (14).

Using a logarithmic interpolation for P and ρ we have

$$\begin{aligned} \ell n\, P(H_i) &= \ell n\, P(Z_i) - \Delta Z[\ell n\, P(Z_{i+1}) - \ell n\, P(Z_i)] \\ &= \ell n\, P(Z_i) - \ell n \left[\frac{P(Z_{i+1})}{P(Z_i)}\right]^{\Delta Z} \end{aligned}$$

or

$$P(H_i) = P(Z_i) \left[\frac{P(Z_{i+1})}{P(Z_i)}\right]^{-\Delta Z}$$

where ΔZ is given by Equation (14).

Similarly

$$\rho(H_i) = \rho(Z_i) \left[\frac{\rho(Z_{i+1})}{\rho(Z_i)}\right]^{-\Delta Z} \tag{16}$$

where ΔZ is again given by Equation (14).

Tabulated Results

Calculations using the interpolation expressions of Equations (13 to 16) are presented in Table 13 for the altitude region of 84 to 121 km′. The values for 84 to 90 km′ provide a means for comparing the results of this calculation with the Standard-Atmosphere values.

Insofar as Equation (12) correctly relates H_i to Z the values of $T_M(H_i)$ would be exactly correct: values of $T(H_i)$, $P(H_i)$ and $\rho(H_i)$, however, would be at best only approximations. Equation (12) however does not accurately represent the relationship between H and Z as used in the U. S. Standard Atmosphere, and the values for the entire Table 13 are correspondingly inaccurate. This situation may be determined by a comparison of the values of P or ρ in Table 13 with those of the Standard Atmosphere for the region 84 to 89 km′. The values of T and T_M for this part of Table 13 do not show the discrepancy because of the isothermal or near isothermal condition in this region. The pressures in Table 13 are seen to be in error by as much as five parts in the fifth significant figure or 0.032 percent when the pressure is 1.5661×10^{-2} mb. The densities which are given to only four significant figures, because they are also limited in the Standard Atmosphere, are seen to vary from the standard values by one part in the fourth significant figure. No test points were available in the standard- atmosphere tabulations for altitudes greater than 90 km′ where the discrepancies are expected to increase. These discrepancies appear to have been caused primarily by the limitation in Equation (12) relating Z to H. If the values of geopotential tabulated in the Standard Atmosphere for integral values of geometric altitude had been given to the nearest hundreth of a meter instead of to only the nearest meter, it appears that interpolation of these values would have been more suitable than using Equation (12) for relating Z to H, and the interpolated values of P and ρ would also have been satisfactory. The lack of such a condition has made it necessary to consider the formal calculation from basic definitions. The chief hurdle to be overcome in such a procedure is the development of an adequate relationship between Z and H.

TABLE 13

VALUES OF T, T_M, p AND ρ FOR SUCCESSIVE ALTITUDE INCREMENTS OF 1000 GEOPOTENTIAL METERS ABOVE 88000 GEOPOTENTIAL METERS WHERE THE VALUES IN THIS TABLE HAVE BEEN DETERMINED BY INTERPOLATION PROCESSES FROM THOSE VALUES TABULATED IN THE U.S. STANDARD ATMOSPHERE IN INCREMENTS OF 1000 GEOMETRIC KILOMETERS, AND WHERE THE RELATIONSHIP BETWEEN GEOPOTENTIAL AND GEOMETRIC ALTITUDE IS EXPRESSED BY EQUATION (12)

H	T	TM	P MB	ρ KG M-3
90.	184.50	184.53	.129978E-02	.24537E-05
91.	187.58	187.61	.108175E-02	.20087E-05
92.	190.63	190.70	.903032E-03	.16496E-05
93.	193.68	193.79	.756010E-03	.13590E-05
94.	196.72	196.88	.634705E-03	.11230E-05
95.	199.74	199.97	.534315E-03	.93080E-06
96.	202.74	203.07	.450989E-03	.77374E-06
97.	205.73	206.16	.381637E-03	.64491E-06
98.	208.69	209.25	.323756E-03	.53901E-06
99.	212.76	213.48	.275427E-03	.44947E-06
100.	217.73	218.64	.235135E-03	.37466E-06
101.	222.67	223.80	.201478E-03	.31365E-06
102.	227.58	228.97	.173251E-03	.26361E-06
103.	232.47	234.13	.149480E-03	.22244E-06
104.	237.32	239.30	.129383E-03	.18837E-06
105.	242.13	244.47	.112342E-03	.16010E-06
106.	246.91	249.64	.978274E-04	.13648E-06
107.	251.67	254.81	.854311E-04	.11680E-06
108.	256.39	259.98	.748095E-04	.10024E-06
109.	265.51	269.67	.657497E-04	.84941E-07
110.	275.26	280.02	.580592E-04	.72236E-07
111.	284.94	290.38	.515001E-04	.61783E-07
112.	294.60	300.74	.458758E-04	.53144E-07
113.	304.20	311.10	.410304E-04	.45947E-07
114.	313.81	321.47	.368310E-04	.39916E-07
115.	323.80	331.84	.331740E-04	.34827E-07
116.	332.75	342.21	.299769E-04	.30521E-07
117.	342.19	352.59	.271702E-04	.26845E-07
118.	353.83	365.29	.247030E-04	.23567E-07
119.	373.23	386.05	.225547E-04	.20359E-07
120.	392.57	406.83	.206918E-04	.17720E-07
121.	411.84	427.61	.190648E-04	.15533E-07
89.	181.44	181.44	.156660E-02	.30079E-05
88.	180.65	180.65	.189194E-02	.36486E-05
87.	180.65	180.65	.228578E-02	.44078E-05
86.	180.65	180.65	.276167E-02	.53252E-05
85.	180.65	180.65	.333663E-02	.64343E-05
84.	180.65	180.65	.403124E-02	.77741E-05

DEVELOPMENT OF EMPIRICAL FUNCTION I RELATING THE NUMERICAL VALUES OF GEOPOTENTIAL AND GEOMETRIC ALTITUDE AS PUBLISHED IN THE UNITED STATES STANDARD ATMOSPHERE

Explicit Function Lacking in Standard Atmosphere

The preceding section, which lists some of the glaring omissions in the U.S. Standard Atmosphere, 1962, indicates that no expression was given whereby the tabulated values of geopotential or other intervening values could be computed from the related geometric altitudes. Neither was there an expression for the reverse process. Accordingly, in view of the need for the expansion of certain tables of the United States Standard Atmosphere it was considered desirable to attempt to develop a relatively simple empirical function suitable for replacing the unavailable and probably highly complicated function used in relating Z to H in the United States Standard Atmosphere, 1962.

One of the simpler approaches involves the use of Equation (12), which when solved for r yields $r(H_i)$ as a function of integral values of H,

$$r(H_i) = \frac{Z}{\frac{g_o}{G}\frac{Z}{H_i} - 1}, \qquad (17)$$

or switching the subscript i from H to Z yields $r(Z_i)$ in terms of integral values of Z.

$$r(Z_i) = \frac{Z_i}{\frac{g_o}{G}\frac{Z_i}{H} - 1} \qquad (18)$$

The introduction of the tabular values of Z_i and the related values of H from the Standard Atmosphere should permit the computation of a set, $r_n(Z)$, of numerical values of r to which r(Z), some analytical function of Z, may be fitted. Then replacing r in Equation (12) with this analytical function one would have the means for computing numerical values of H as a function of Z_i with results comparable to those in the Standard Atmosphere:

$$H(Z_i) = \frac{Z_i\, r(Z_i)}{r(Z_i) + Z_i}\,\frac{g_o}{G} \qquad (19)$$

or

$$H(Z_i) = \frac{Z_i\, g_o/G}{1 + [Z_i/r(Z_i)]} \qquad (20)$$

Similarly, one could in principle develop an empirical analytical expression for $r(H)$. With this function, numerical values of Z as a function of H_i could be computed from the expression

$$Z(H_i) = \frac{H_i}{(g_o/G) - [H_i/r(H_i)]} \tag{21}$$

Since the standard-atmosphere tables above 120 km are given only in terms of Z_i with related values of H, the present discussion is limited to the problem of developing $r(Z_i)$.

The relationship between Z and H as used by Minzner [1] in the ARDC Model Atmosphere may be expressed by Equations (19), (20) or (21) in which

$$r(Z_i) = r(H_i) = r(\Phi_1) = 6{,}356{,}766 \text{ m}$$

where Φ_1 represents the latitude 45° 32' 33".

Using the Lambert theory as described by Harrison [14] on which these equations are based, one may show that seven significant figures are required to specify that value of r which is consistent with the latitude associated with 9.80665 m $\sec^{-2}$, the defined sea-level gravity acceleration. The relationship between Z and H as used in the Standard Atmosphere 1962 represents a refinement of the Lambert Theory. One may expect, therefore, that numerical values of $r(Z_i)$ from Equation (18) using the data from the Standard Atmosphere 1962 should also be carried to seven significant figures if these values are to be meaningful. On the basis of its association with the Lambert theory one might also expect the value of $r(Z_i)$ for $Z_i = 0$ to be close to 6,356,766 meters.

Considering standard-atmosphere pairs of values of Z and H ranging from six significant figures at the greatest altitudes down to 1 significant figure at sea level, it is immediately apparent that it is impossible to obtain from Equation (18) numerical values of r which vary smoothly in the seventh significant figure over any part of the altitude range involved. Nevertheless, 500 values of $r(Z_i)$ were computed, one for each successive integral multiple of one kilometer from 1 to 300 km and one value for each successive integral multiple of two kilometers from 300 to 700 km. The reliable number of significant figures in the calculated values of $r(Z_i)$ vary widely for various values of Z. For the altitude region 650 to 700 km, the successive values of r had differences ranging from -8 to +7 units in the sixth significant figure. In the altitude region of 67 to 70 km the successive values of r had differences ranging from -4 to +4 units in the fourth significant figure. In the altitude region of 5 to 10 km the successive differences ranged between +2 and -2 in the second significant figure. It appears that the uncertainty is approximately proportional to some power function of the reciprocal of the altitude. The scatter in the computed values of r made these values useless when determined for small values of Z and H. In an attempt to smooth these erratic results, each group of five consecutive values of r were averaged yielding 100 average values which are plotted in Figure 11. Below 100 km, the mean values of r are so scattered that they are almost useless in the determination of a function $r(Z)$.

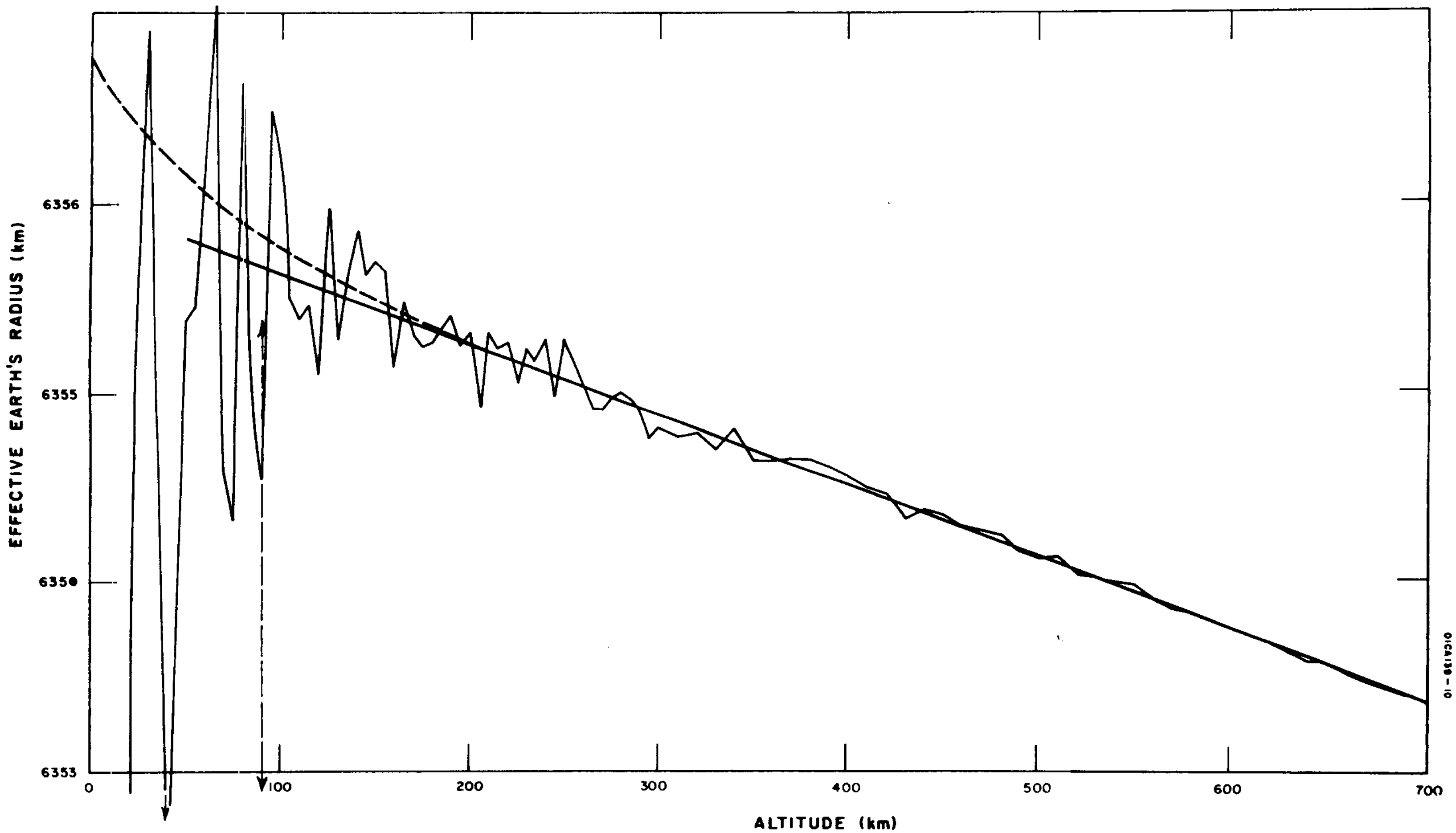

Figure 11. Values of effective earth's radius r which when used in Equation (19) with the corresponding values of Z yield values of geopotential approximately equal to those given by the U.S. Standard Atmosphere 1962 for those values of Z.

A first-degree curve and a second-degree curve were fitted to those mean values of r for $Z \geq 50$ km. These curves were so similar that their differences would not be detectable on the graph and the single solid line in Figure 11 represents both functions. The dashed line represents a subjective estimate of the function r(Z) based on the previously-mentioned value 6,356,766 m for Z = 0. The data contain little real basis for this estimate, however, or for any other estimate in the 0 to 50 km region.

An error analysis of r as determined by Equation (18) indicates that

$$\frac{\delta r}{r} = \frac{Z}{Z - \frac{H g_o}{G}} \frac{\delta H}{H} \tag{22}$$

For Z between 1.8 and 3.15 km, the difference $Z-(Hg_o/G)$ as tabulated in the Standard Atmosphere is one meter, while δH the uncertainty in H varies from +0.50 m′ to -0.49 m′. Since the quotient Z/H is very nearly unity, it is apparent that the uncertainties in H represent an uncertainty in r of approximately +50 and -50 percent, respectively. In order to obtain a value of r certain to the seventh significant figure or to ± 0.5 meters for this altitude region, it would be necessary for H to be known to 1×10^{-7} meters or to eleven significant figures. For Z = 700 km, however, H needs to be known to only ± 0.5 meters or to seven significant figures in order that the related value of r be known to the same relative uncertainty.

Because of the limitations of the existing data this method for developing a simple Z to H relationship consistent with the United States Standard Atmosphere 1962 is useless. If the original standard-atmosphere calculations could be reworked to the required accuracy of H, these new data could serve as the basis of a simplified relationship. With only the existing data, however, another approach must be employed.

DEVELOPMENT OF EMPIRICAL FUNCTION II RELATING THE NUMERICAL VALUES OF GEOPOTENTIAL AND GEOMETRIC ALTITUDE AS PUBLISHED IN THE UNITED STATES STANDARD ATMOSPHERE

Comparison of Different Sets of Calculations of Geopotential

Geopotential in the 1956 ARDC Model Atmosphere [1] was calculated by the expression

$$H_{56} = \frac{rZ}{r+Z} \cdot \frac{g_o}{G} \tag{23}$$

where r = 6,356,766 meters, and where H_{56} is used to differentiate the results of this particular calculation of geopotential from the values tabulated in U.S. Standard Atmosphere, for which values the designation is H_{62}.

In the 1962 U.S. Standard Atmosphere, geopotential H_{62} at any altitude Z was computed by the integration of a complicated gravity-acceleration expression along a path identical to the curved line of gravitational force passing through the point in question (where the point in question was defined relative to geometric latitude and relative to the distance from the center of an ellipsoid rather than relative to sea level) such that for each successive altitude, the integration must be performed along a different line of force. No specific equation suitable for direct numerical evaluation of H_{62} as a function of Z was given in the standard-atmosphere document, nor has one been developed from the fundamental considerations which were given. Instead, a simple approximation formula in the nature of Equation (23) with a correction term was developed; i.e.,

$$H_{62/8} = \frac{rZ}{r+Z} \cdot \frac{g_o}{G} - f(Z) \tag{24}$$

where $H_{62/8}$ represents the eight-significant-figure values which when rounded lead to $H_{62/R}$, the values of geopotential published in the Standard Atmosphere.

A comparison of the tabulated six-significant-figure values (above 100,000 m) of H_{62} with $H_{56/8}$ the eight-significant-figure values of H_{56} as obtained from Equation (23) suggest the following observations:

(1) The tabulated values of H_{62} are always less than the corresponding value of $H_{56/8}$ for altitudes greater than sea level.

(2) The tabulated values of H_{62} are rounded from original seven- or eight-significant-figure values. (These rounded values will henceforth be designated as $H_{62/R}$).

(3) The values of $H_{62/8}$ which have been rounded to $H_{62/R}$ would most likely differ from $H_{56/8}$ in accordance with f(Z), some smooth monotonically increasing function of Z, which function has a value of zero at sea

level and a value of about 33 geopotential meters (m′) at the geometric altitude Z of 700 km ; i.e.,

$$f(Z) = H_{56/8} - H_{62/8} \qquad (25)$$

Thus, a curve fit to the difference between $H_{56/8}$ and $H_{62/8}$ (if these were available) would provide the desired function f(Z), and the desired approximation expression for calculating $H_{62/8}$ as in Equation (24) would have been determined. Unfortunately, values of $H_{62/8}$ are not available, and an indirect approach must be pursued.

An Approach for Generating f(Z)

An imaginary set of eight-significant-figure values of H_{62} is hypothesized. If the hypothetical numerical values of $H_{62/8}$ were compared with Z in the region of sea level and immediately above, one would find that at Z = 0, $H_{68/8} = 0$, and as Z increases above zero, $H_{62/8}$ would also increase but at a lesser rate than Z, so that at $Z = Z_{0.4999}$, which is the symbol for a specific altitude located somewhere between Z = 1750 and Z = 1800 meters, the value of $H_{62/8}$ would lag behind that of Z by exactly 0.4999 of a meter. At $Z = Z_{0.5000}$ which is the symbol for an altitude immediately above $Z_{0.4999}$, the value $(Z-H_{62/8})$ would become 0.5000. Between Z = 0 and Z = 4000 m, the value of $(Z-H_{62/8})$ would increase smoothly as Z increases in accordance with the values presented in Table 14.

The exact numerical values of the symbolic altitudes $Z_{0.4999}$, $Z_{0.5000}$, $Z_{1.4999}$, etc. are not known, but from the U.S. Standard Atmosphere 1962 we may infer that these values are bounded within particular limits indicated in Table 14. Thus $Z_{0.4999}$ and $Z_{0.5000}$ have values between 1750 and 1800 meters, while $Z_{1.4999}$ and $Z_{1.5000}$ would be found between the altitudes 3050 and 3100 meters, etc.

Returning momentarily to the reality of the U.S. Standard Atmosphere 1962, we can examine the difference between the tabulated integral values of geometric altitude Z_i and the rounded values of geopotential altitude $H_{62/R}$ where Z_i is some integral multiple of 1 meter. One finds that the difference $(Z_i - H_{62/R})$ increases discontinuously in integral steps of one meter as Z increases, with successive discontinuties occurring between $Z_{0.4999}$ and $Z_{0.5000}$, and again between $Z_{1.4999}$ and $Z_{1.5000}$, etc. as indicated in Table 14. The differences $(Z-H_{62/8})$ are also tabulated.

Two differences given in Table 14 are shown in Figure 12 where the hypothetical quantity $(Z - H_{62/8})$ is shown as the solid-line, smooth-curve function, and the realistic quantity $(Z_i - H_{62/R})$ is shown as the discontinuous function represented as a series of alternate horizontal and vertical line segments, where these line segments connect the series of discrete points derived from the finite number of tabulated values. A graph of the difference $(Z - H_{56/8})$ is also shown as a smooth, dashed-line curve in Figure 12 where $H_{56/8}$ represents the eight-significant-figure values obtained from Equation (23).

TABLE 14

DIFFERENCES $(Z - H_{62/8})$ and $(Z_i - H_{62/R})$ AS A FUNCTION OF ALTITUDE

Numerical Altitude (meters)	Symbolic Altitude	Difference $(Z - H_{62/8})$ (meters)	Difference $(Z_i - H_{62/R})$ (meters)
0	Z_0	0.0000	0
1750			0
	$Z_{0.4999}$	0.4999	0
	$Z_{0.5000}$	0.5000	1
1800			1
3050			1
	$Z_{1.4999}$	1.4999	1
	$Z_{1.5000}$	1.5000	2
3100			2
3950			2
	$Z_{2.4999}$	2.4999	2
	$Z_{2.5000}$	2.5000	3
4000			3

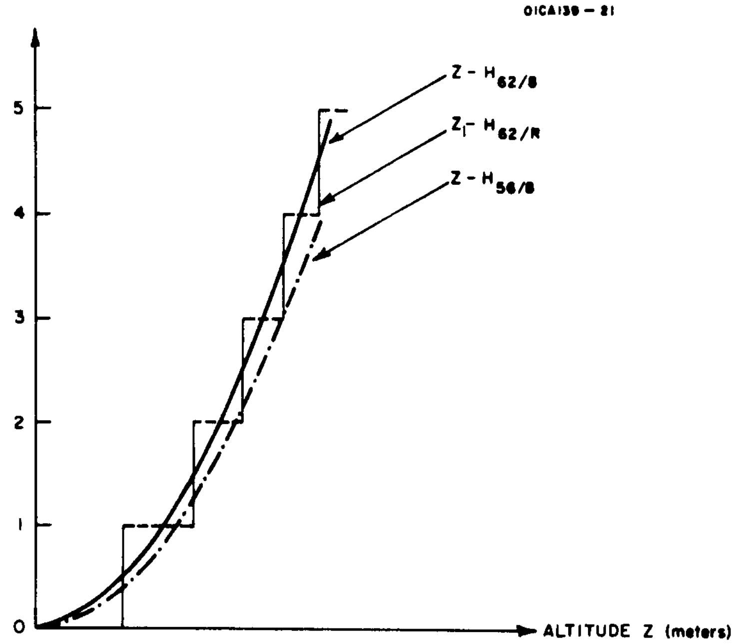

Figure 12. Differences $Z - H_{62/8}$, $Z_i - H_{62/R}$ and $Z - H_{56/8}$ vs altitude.

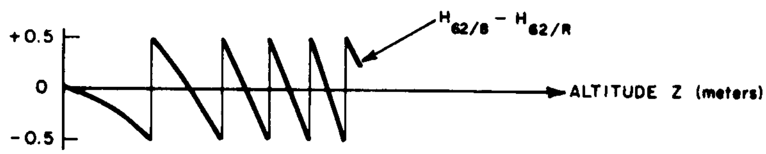

Figure 13. Difference $H_{62/8} - H_{62/R}$ vs altitude.

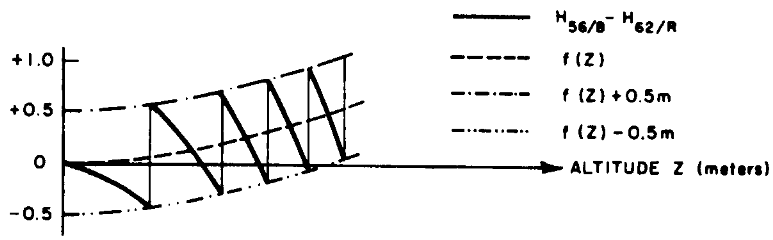

Figure 14. Difference $H_{56/8} - H_{62/R}$, as well as $f(Z)$, $f(Z) + 0.5$ m, and $f(Z) - 0.5$ m all as a function of altitude.

Since $Z - H_{56/8}$ is always less than $Z - H_{62/8}$, it is apparent that $H_{56/8}$ departs from Z less rapidly than does $H_{62/8}$, and the difference $(H_{56/8} - H_{62/8}) = f(Z)$ increases from zero as Z increases from zero.

If the graphs of Figure 12 were extended to include the region from 0 to 90 kilometers altitude, the graph of $(Z_i - H_{62/R})$ would show 1257 vertical line segments representing the same number of abrupt discontinuities while, if extended to include the region from 90 to 700 kilometers altitude, the graph would include an additional 68,216 such discontinuities. In these same two altitude intervals a graph (not presented in Figure 12) showing the difference between Z and rounded values of H_{56}, i.e., $(Z_i - H_{56/R})$ would show one less and 33 less discontinuities, respectively, than would be seen in the extended graph of $Z_i - H_{62/R}$.

If the hypothetical graph $(Z - H_{62/8})$ and the realistic graph $(Z_i - H_{62/R})$ are compared, we find that at the particular points for which Z is an integral multiple of one meter, $Z - Z_i = 0$, and the difference $H_{62/8} - H_{62/R})$ varies between +0.5 and about -0.5 meter in accordance with the graph of Figure 13. A careful examination shows that the limiting differences at the points of discontinuity are separated by values of 0.9999 m′, 0.999 m′ or 0.99 m′, consistent with the fixed eight significant figures in $H_{62/8}$ and the varying number in $H_{62/8}$ depending upon the altitude region. It is also apparent that these discontinuities are symmetrical about the horizontal axis.

A comparison of the discontinuous graph $(Z_i - H_{62/R})$ and the continuous graph $(Z - H_{56/8})$ at those points for which Z has an integral value yields a difference $(H_{56/8} - H_{62/R})$ which follows the pattern of Figure 14. In this figure, the discontinuities occur at the same altitudes as for Figure 13. The limiting differences at the points of discontinuity are the same as for the corresponding discontinuity of Figure 13. Contrary to the situation in Figure 13, these limiting points are not symmetrical about the horizontal axis but are symmetrical about the curved-line function $f(Z)$ which is the function being sought for use in Equation (24).

Unfortunately, the number of values of $H_{62/R}$ in the U.S. Standard Atmosphere is not sufficient so that a graph of these points would show the detailed type of pattern of Figure 14: there are only 420 values of $H_{62/R}$ for altitudes between 90 to 700 km compared with 68216 regions of discontinuity for that same range of altitudes. Consequently, any single value $(H_{56/8} - H_{62/R})$ from available data represents but a single point on a section of graph which, on the average, might have 170 regions of discontinuity. Obviously the detailed step-function graph cannot be produced from the available data. On a graph in which the altitude scale has been sufficiently compressed, however, a plot of the available 420 data points (above 90 km) appears as a band of randomly scattered points in which the extreme ordinate values for any single value of abscissa may never exceed a difference corresponding to one geopotential meter, as in Figure 14. Hopefully, a sufficient number of data points lie near enough to the two extremes to permit the establishment of a good approximation to the locus of the upper and lower boundary of the 1-m′ band.

The smoothed lower bound of the envelope of the set of scattered data $(H_{56/8} -, H_{62/R})$ represents a subset of values which are never more than 0.4999 m$'$ below the desired function $(H_{56/8} - H_{62/8}) = f(Z)$. Similarly, the smoothed upper bound of the envelope of the scattered data represents a set of values which are never more than 0.5 m$'$ greater than the desired $f_1(Z)$. Thus, if 0.4999 m$'$, 0.499 m$'$, or 0.49 m$'$ is added to the individual values of the difference set $(H_{56/8} - H_{62/R})$, each in the appropriate altitude region, a set of data called Source Set 1 is formed. The smoothed lower bound of the values of Source Set 1 consist of a smaller set of data called Subset 1. The points of Subset 1 will have values equal to or very slightly greater than the values of the desired function f(Z) at the corresponding altitudes.

Similarly if 0.5000 m$'$ is subtracted from the individual values of the difference set $(H_{56/8} - H_{62/R})$, to form Source Set 2, the smoothed upper bound of these downwardly adjusted data form a set of points called Subset 2. These data points of Subset 2 will have values equal to or slightly less than the values of the desired function f(Z).

The points of Subset 1 and Subset 2 are then subjected to a further graphical selection to form Smoothed Subsets 1 and 2. The points of these Smoothed Subsets 1 and 2 are then combined, and when processed in a curve-fitting program determine a close approximation to the desired function f(Z). The numerical and graphical processes employed depend upon the assumption that the function f(Z) as well as its first derivative are monotonically increasing with Z, a situation which is apparently true for the upper and lower bounds of the band of data points.

The detailed steps of the process are as follows:

(1) Prepare Source Set 1 by performing the following operations as appropriate:

add 0.4999 to $(H_{56/8} - H_{62/R})$ for $1.0000 \text{ km} \leq Z \leq 9.9999 \text{ km}$,

add 0.499 to $(H_{56/8} - H_{62/R})$ for $10.000 \text{ km} \leq Z \leq 9.9999 \text{ km}$,

add 0.49 to $(H_{56/8} - H_{62/R})$ for $100.00 \text{ km} \leq Z \leq 9.9999 \text{ km}$,

(2) Prepare Subset 1 as follows:

(a) Scan the entire Source Set 1 for the smallest positive number (all of the numbers of this set will be positive), and store this value with its corresponding value of Z as the first entry of Subset 1.

(b) Remove this value and those for lower altitudes from Source Set 1 and discard.

(c) Scan the remaining members of Source Set 1 for the lowest value and store this value with its corresponding value of Z as the 2nd entry of Subset 1.

(d) Remove this member along with those associated with lower altitudes from Source Set 1. Repeat steps (c) and (d) until all the values of Source Set 1 have been removed, and Subset 1 has been developed. If there are two or more lowest values at any of the above steps store only that one corresponding to the greatest altitude, and reject the others.

(3) Punch and print the stored values of Subset 1.

(4) Prepare Source Set 2 by performing the following operation for all altitudes of interest:

$$\text{subtract } 0.5000 \text{ from } (H_{56/8} - H_{62/R})$$

(5) Prepare Subset 2 as follows:

(a) Scan the entire Source Set 2 for negative values, which will be found at the low-altitude end of the set, removing and discarding these members of the set.

(b) Scan the remainder of Source Set 2 for the largest positive value and store this value along with its associated altitude value as the first member of Subset 2.

(c) Remove this member from Source Set 2 along with all members associated with greater altitudes.

(d) Repeat steps (b) and (c) until all members of Source Set 2 have been removed and Subset 2 has been developed. If there are two or more greatest values at any point in the scanning operation, store only that one associated with the lowest altitude and discard the others.

(6) Punch and print the stored values of Subset 2.

(7) Plot the data points of Subset 1 on large-scale graphs (not shown in this report) for further data selection. The suggested scales for these graphs are indicated in Table 15.

(8) Select certain points of Subset 1 which appear to form a smooth monotonically increasing lower bound to the total of Subset 1 data. These selected points comprise Smooth Subset 1 and are those points which may be connected sequentially with straight-line segments meeting the following two conditions:

(a) Each of these line segments lies below all those points in Subset 1 having altitude values within the altitude interval encompassed by the particular line segment.

(b) The successive line segments have slopes which are monotonically increasing for increasing altitudes.

TABLE 15

SCALE VALUES OF $f_1(Z)$ AND Z EMPLOYED IN DIFFERENT PORTIONS OF THE GRAPHS OF SUBSETS 1 AND 2

Altitude Interval km	Meters of $f_1(Z)$ Per 1 cm of Graph	km of Altitude Z Per 1 cm of Graph
0 - 150	0.02	4
100 - 250	0.04	4
220 - 370	0.10	4
360 - 550	0.10	4
550 - 700	0.10	1

The series of straight-line segments meeting these conditions is designated as f(SS-1). The ordinate values of the desired function f(Z) may be equal to or less than the ordinate values of the end points of the segments of f(SS-1), but will always be less than the ordinate values of all other parts of f(SS-1).

(9) Plot the data from Subset 2 on the same graphs with Subset 1.

(10) Select certain points of Subset 2 which appear to form a smooth monotonically increasing upper bound to the total of Subset 2 data. These selected points comprise Smooth Subset 2 and are those points which may be connected sequentially with straight-line segments meeting the following two conditions:

(a) Each of these line segments lies above all those points of Subset 2 having altitude values within the altitude interval encompassed by the particular line segment.

(b) Same as condition b under step 8.

This series of straight-line segments associated with Smooth Subset 2 data is designated as f(SS-2) and will be seen to be close to but below the segments f(SS-1) prepared in Step 8. The ordinate values of the desired function f(Z) may be equal to or greater than the corresponding ordinate values of f(SS-2) for all values of Z, but there is a small probability that the ordinate values of f(Z) may sometimes be slightly less than the corresponding ordinate values of f(SS-2) for some values of Z in between the end points of some of the line segments, i.e., the points of smooth Subset 2. For any particular set of abscissa values Z_2 corresponding to the abscissa of the data points in Smooth Subset 2, the desired smooth function f(Z) has ordinate values which are equal to or greater than the ordinate values of the related points of Subset 2, but which are less than the corresponding ordinate values of f(SS-1). Thus, the value of f(Z) corresponding to each member of the set of ordinates Z_2 is bounded within small limits, and it is possible to estimate the value of f(Z) for each value of Z_2 to be midway between the appropriate limits. It is also possible to estimate the value of δf(Z), the range of uncertainty of f(Z), to be equal to the separation between the above specified boundaries.

Values of f(Z) and δf(Z) may similarly be made for the set of abscissa values Z corresponding to the abscissas of the data points in Smooth Subset 1. In these instances, however, the ordinate values of f(Z) will be equal to or less than the ordinate values of the related data points of Smooth Subset 1, and may be as low as or even slightly lower than the ordinate value of the corresponding points of f(SS-2).

(11) From the graphical representations f(SS-1) and f(SS-2) connecting the data points of Smooth Subsets 1 and 2 respectively, estimate values of f(Z) and δf(Z) for the abscissas in the set Z_1 and Z_2 in accordance with the discussion under Step 10.

The set of graphically estimated values of f(Z) and uncertainty of f(Z) in the form of 100 $\delta f(Z)/f(Z)$ are presented as a function of Z in Table 16.

The f(Z) Polynomial

The data of Table 16 were fed into a digital-machine curve-fitting program designed to yield the coefficients of best-fit, first-, second-, third-, and fourth-degree polynomials as well as the difference between each point of input data and the polynomial values for the same abscissas. From considerations of mean differences, the fourth-degree curve was found to give the best fit of the four curves considered. For the case when Z is expressed as kilometers, we find

$$f(Z) = A + BZ + CZ^2 + DZ^3 + EZ^4 \tag{26}$$

where f(Z) is expressed as m'

$$A = 0.4858124 \times 10^{-2} \ m'$$

$$B = 0.1338918 \times 10^{-3} \ m'/km$$

$$C = 0.1903029 \times 10^{-4} \ m'/km^2$$

$$D = 0.8288881 \times 10^{-9} \ m'/km^3$$

$$E = 0.1822113 \times 10^{-10} \ m'/km^4$$

When Z is expressed in meters, the values of the several coefficients are multiplied by $(10^{-3})^x$ $(km/m)^x$, where x is the power of Z in the term with which the particular coefficient is associated. In both instances the dimensions of f(Z) is geopotential meters. A list of the functional values of f(Z) as defined above and a list of the difference between the graphical and functional value of f(Z) are also given in Table 16.

It is noted that the function determined does not pass exactly through zero at Z = 0, but has a value of 0.004858124 meter at this altitude. This value, in effect, increases the value of the entire function by less than five thousandths of a meter, an amount which is trivial in the determination of geopotential to the nearest hundreth of a meter. This discrepancy undoubtedly results because the graphically derived values of f(Z) have only limited accuracy. In addition, only three graphical values of f(Z) were determined for the region 0 to 90 km in which region the initial study considered only those 90 tabulated values of Z for integral multiples of one km'. There are, however, approximately 600 tabular entries of Z and $H_{62/R}$ between 0 and 90 km', and more than three graphical values of f(Z) could have been obtained for that region. A better fit at Z = 0 might have been determined if all the available data had been used. No great error is made, however, if zero is assigned to the coefficient of the Z^o term.

TABLE 16

GRAPHICALLY DETERMINED ESTIMATES OF $f_1(Z)$ COMPARED WITH POLYNOMIALLY DETERMINED VALUES OF $f(Z)$

Z	Graphical Estimates Of $f_1(Z)$	Percentage Degree of Certainty	Functional Value of $f(Z)$	Graphical Value Minus Functional Value
.000E-50	.00000E-50	100.000	.4858124E-02	-.4858124E-02
.440E+02	.42000E-01	90.576	.4280204E-01	-.8020470E-03
.720E+02	.13000E+00	96.923	.1243193E+00	.5680610E-02
.112E+03	.34200E+00	98.830	.3421639E+00	-.1639800E-03
.164E+03	.86000E+00	97.680	.8471755E+00	.1282441E-01
.203E+03	.14500E+01	98.620	.1424355E+01	.2564490E-01
.215E+03	.16450E+01	98.480	.1640593E+01	.4407000E-02
.225E+03	.18210E+01	98.050	.1835597E+01	-.1459790E-01
.245E+03	.22550E+01	99.334	.2267670E+01	-.1267070E-01
.258E+03	.25700E+01	99.222	.2579805E+01	-.9805100E-02
.266E+03	.27750E+01	99.097	.2784586E+01	-.9586400E-02
.273E+03	.29550E+01	98.815	.2971894E+01	-.1689450E-01
.372E+03	.65100E+01	99.538	.6506622E+01	.3377400E-02
.456E+03	.11010E+02	99.728	.1097247E+02	.3752700E-01
.486E+03	.12925E+02	99.652	.1293304E+02	-.8047000E-02
.492E+03	.13360E+02	99.776	.1334955E+02	.1045000E-01
.518E+03	.15265E+02	99.771	.1525077E+02	.1422500E-01
.530E+03	.16175E+02	99.725	.1618200E+02	-.7009000E-02
.570E+03	.19510E+02	99.897	.1953848E+02	-.2848800E-01
.612E+03	.23450E+02	99.787	.2349453E+02	-.4453200E-01
.642E+03	.26650E+02	99.625	.2660026E+02	.4973800E-01
.676E+03	.30405E+02	99.852	.3041133E+02	-.6333000E-02

Application of f(Z) to the Determination of Geopotential for Z = 90 km

The function f(Z) when used in Equation (24) provides the means for computing the value of $H_{62/8}$ for specified values of Z. An evaluation of Equation (26) for Z = 90 km shows f(Z) to be 0.201 m$'$ when the coefficient A is taken to be zero. The corresponding value of H from Equation (24) is found to be 88743.335 m$'$. This value should be a close approximation to that value of geopotential at which the defining temperature-altitude profiles of the U.S. Standard Atmosphere 1962 are switched from being linear in terms of geopotential, in the lower regime, to being linear in terms of geometric altitude in the upper regime. The calculation of the pressure for this transition altitude is considered under the discussion related to pressures at critical altitudes at 90 and below 90 km in the next section.

Application of f(Z) Data to the Determination* of an Expression for Z versus H

The calculation of geometric altitude in terms of integral multiples of one geopotential kilometer for the required tables for the upper regime of the Standard Atmosphere may not be made by means of Equation (24), involving the functional expression for f(Z), without an undesirable iteration process. If Equation (24) is solved for Z without considering f(Z) explicitly in terms of Z, one obtains

$$Z = \frac{r[H + f(Z)]}{r - [H + f(Z)]} \quad . \tag{27}$$

Since f(Z) represents a set of values associated with a specific set of geometric altitudes, it is apparent that this same set of values may become f(H) by being associated with a specific set of geopotentials, related to the geometric altitude through Equation (24). With this transformation of f(Z) to f(H), Equation (27) becomes

$$Z_{62/8} = \frac{r[H + f(H)]}{r - [H + f(H)]} \tag{28}$$

The function f(H) is found from the same basic graphical data employed in finding f(Z). In this case the value -0.342 m$'$, for example, previously associated with 112,000 geometric meters in Table 16 is now associated with 110,060.488 geopotential meters, i.e.,

$$\frac{rZ}{r+Z} - 0.342 = 110060.83 - 0.342 = 110,060.488 \text{ m}' \quad .$$

Similar relationships apply for each of the other data points. These revised data points presented in Table 17 yield a new polynomial fit,

$$f(H) = AA + ABH + ACH^2 + ADH^3 + AFH^4 \quad , \tag{29}$$

*The development of Equations (27) through (29) and the Table 17 form the basis for the geopotential tables included in the U.S. Supplementary Atmos. (1966).

TABLE 17

GRAPHICALLY DETERMINED ESTIMATES OF f_1(H)
COMPARED WITH POLYNOMIALLY DETERMINED VALUES OF f_1(H)

Geopotential	Graphical Estimates of f_1(H)	Percentage Degree of Certainty	Functional Value of f_1(H)	Graphical Value Minus Functional Value
.0000000E-50	.00000E-50	100.000	.2579651E-02	-.2579651E-02
.4369749E+02	.42000E-01	95.576	.4386026E-01	-.1860263E-02
.7119349E+02	.13000E+00	96.923	.1262009E+00	.3799100E-02
.1100604E+03	.34200E+00	98.630	.3441282E+00	-.2128220E-02
.1598744E+03	.86200E+00	97.680	.8482695E+00	.1373044E-01
.1967164E+03	.14500E+01	96.620	.1424622E+01	.2537790E-01
.2079644E+03	.16450E+01	98.480	.1640626E+01	.4373300E-02
.2173064E+03	.18210E+01	98.050	.1835452E+01	-.1445220E-01
.2359054E+03	.22550E+01	99.334	.2267215E+01	-.1221530E-01
.2479344E+03	.25700E+01	99.222	.2579188E+01	-.9188600E-02
.2553134E+03	.27750E+01	99.097	.2783888E+01	-.8888100E-02
.2617554E+03	.29550E+01	98.815	.2971135E+01	-.1613520E-01
.3514274E+03	.65100E+01	99.538	.6505996E+01	.4003100E-02
.4254674E+03	.11010E+02	99.728	.1097266E+02	.3733800E-01
.4514694E+03	.12925E+02	99.652	.1293344E+02	-.8443000E-02
.4566424E+03	.13360E+02	99.776	.1334997E+02	.1002700E-01
.4789544E+03	.15265E+02	99.771	.1525125E+02	.1374500E-01
.4891954E+03	.16175E+02	99.725	.1618248E+02	-.7483000E-02
.5230754E+03	.19510E+02	99.897	.1953878E+02	-.2878400E-01
.5582304E+03	.23450E+02	99.787	.2349426E+02	-.4426000E-01
.5830824E+03	.26650E+02	99.625	.2659992E+02	.5007600E-01
.6109914E+03	.30405E+02	99.852	.3041092E+02	-.5925000E-02

where $f(H)$ is expressed as m′

$$AA = 0.2579651 \times 10^{-2} \quad m'$$
$$AB = 0.2161710 \times 10^{-7} \quad m'/m'$$
$$AC = 0.1807561 \times 10^{-10} \quad m'/(m')^2$$
$$AD = 0.9153012 \times 10^{-16} \quad m'/(m')^3$$
$$AE = 0.2006785 \times 10^{-22} \quad m'/(m')^4$$

and where H is the altitude in geopotential meters. If H is to be expressed in kilometers, the values of the various coefficients must be multiplied by $(10^3)^x \, (m'/km')^x$ where x is the power to which H is raised in that term to which the coefficient applies.

Equation (29) introduced into Equation (28) now yields values of Z for integral values of H in accordance with the related values tabulated in the U.S. Standard Atmosphere 1962. The application of this equation to the expansion of the Standard Atmosphere above 90 km is considered below, under the discussion of pressure and density tables.

PROBLEMS RELATED TO THE RECALCULATION OF PRESSURE AT CRITICAL ALTITUDES BELOW 90 AND AT 90 km

Standard Atmosphere Pressures Not Reproducible Using Standard Values of Constants

To obtain a seven-significant-figure value of p for Z = 90,000 m it is necessary to begin with the following basic definitions of the Standard Atmosphere: (1) the sea-level value of pressure, p_o = 1013.250 mb, (2) the sea-level value of temperature T_{M_o} = 288.15°K, (3) the values of L_M' the gradients of T_M relative to geopotential for the several atmospheric layers, and (4) the values of the several constants G, M_o, R^*. One may then compute the values of p and T_M at each of the critical altitudes (i.e., at the following geopotential altitudes: 11, 20, 32, 47, 53, 61, and 79 geopotential kilometers) and at 88743.355 m′, that value of the geopotential equivalent to 90,000 geometric meters previously deduced. These calculations are performed by the simple equations:

$$\frac{p}{p_b} = \left[\frac{(T_M)_b}{(T_M)_b + L_M' \quad (h-h_b)}\right]^{-Q/L_M'} \tag{30}$$

and

$$\frac{p}{p_b} = \exp\left[-Q\,\frac{(h-h_b)}{(T_M)_b}\right] \tag{31}$$

depending upon whether the constant temperature gradient for any given layer is non-zero or zero.

The numerical value of Q is determined by the fraction $\frac{GM_o}{R^*}$ where those three quantities are defined as follows:

$$G = 9.80665 \text{ m}^2\text{sec}^{-2}\ (\text{m}')^{-1}$$
$$M_o = 28.9644 \text{ kg (kmole)}^{-1}$$
$$R^* = 8.31432 \times 10^3 \text{ Joules } (^\circ\text{K})^{-1}\ (\text{kmole})^{-1}$$

The value of each of these three quantities is given to six signigicant figures, and the value of Q used in any calculation depends to some extent upon how these quantities are combined and rounded. If each step is carried to nine or more significant figures and rounded to eight significant figures the resulting value of Q is 0.034163195. If each step is carried to eight or seven significant figures and rounded to seven or six significant figures respectively, the values of Q differ in the sixth significant figure. The values of Q for all three cases are:

9 significant figures rounded to 8	0.034163195
8 significant figures rounded to 7	0.03416319
7 significant figures rounded to 6	0.0341632

Values of pressures computed for the various levels using the first and third of these values of Q with Equations (30) and (31) yielded values of pressure which were not in agreement with those of the U.S. Standard Atmosphere at critical altitudes 20, 32, 47, 53, and 61 km'. It was obvious therefore that the computations of p for the Standard Atmosphere involved a value of Q larger than any of three cited above, and that value of Q must be found if standard-atmosphere calculations are to be duplicated.

Determination of that Value of Q Which Yields Standard-Atmosphere Values of Pressure

Recognizing that digital computers are sometimes programmed to chop a result after a specified number of digits, and sometimes to round, 18 values of Q were determined, one for each of three possible combinations of orders of operations of the quantities G, M_0, and R^*, applied to six different numbers of significant figures or rounding procedures as illustrated in Table 18. From this table, it appears that while the more probable values of Q are those previously considered, depending upon the number of significant figures carried, other possible but rather improbable values of Q from the computer might be 0.03416322 and 0.03416323. Consequently, sets of values of pressure were computed for each of the seven critical altitudes not including the geopotential equivalent of 90 geometric kilometers (which at that time was not known) using each of the following seven arbitrary values of Q.

Q1 = 0.034163195

Q2 = 0.03416320

Q3 = 0.03416321

Q4 = 0.03416322

Q5 = 0.03416323

Q6 = 0.034163235

Q7 = 0.03416324

The resulting pressures are listed in Table 19 without the appropriate power of 10 (for reasons of space) since only the significant figures are of interest in this comparison. When the first six significant figures of the computed pressure for a particular critical altitude agree respectively with the digits of the corresponding standard-atmosphere value (in a row labeled STD), these six significant figures only are underlined.

TABLE 18

VALUES OF Q AS COMPUTED FROM SEVERAL COMBINATIONS OF ORDERS OF PROCEDURE AND FOR SUCCESSIVE CUTTING OR ROUNDING NUMBERS OF SIGNIFICANT FIGURES

	(g_o/R^*)			$Q = (\frac{g_o}{R^*})\, M_o$
cut at 10 digits	0.001179489122	x M_o	=	0.03416319472
cut at 9 digits	0.00117948912	x M_o	=	0.03416319466
cut at 8 digits	0.0011794891	x M_o	=	0.03416319408
cut at 7 digits	0.001179489	x M_o	=	0.03416319119
cut at 6 digits	0.00117948	x M_o	=	0.03416293051
rounded to 6 digits	0.00117949	x M_o	=	0.03416322
	(g_o/M_o)			$Q = \frac{(g_o\, M_o)}{R^*}$
cut at 10 digits	284.0437332	÷ R*	=	0.03416319472
cut at 9 digits	284.043733	÷ R*	=	0.03416319470
cut at 8 digits	284.04373	÷ R*	=	0.03416319434
cut at 7 digits	284.0437	÷ R*	=	0.03416319073
cut at 6 digits	284.043	÷ R*	=	0.03416310654
rounded to 6 digits	284.044	÷ R*	=	0.03416322681
	(M_o/R^*)			$Q = (\frac{M_o}{R^*})\, g_o$
cut at 10 digits	0.003483676355	x g_o	=	0.03416319472
cut at 9 digits	0.00348367635	x g_o	=	0.03416319467
cut at 8 digits	0.0034836763	x g_o	=	0.03416319418
cut at 7 digits	0.003483676	x g_o	=	0.03416319124
cut at 6 digits	0.00348367	x g_o	=	0.03416313240
rounded to 6 digits	0.00348368	x g_o	=	0.03416323047

TABLE 19

STANDARD-ATMOSPHERE VALUES OF PRESSURE (WITHOUT THE FACTOR OF THE APPROPRIATE POWER OF TEN) AT EACH OF SEVEN CRITICAL ALTITUDES, COMPARED WITH PRESSURES AT THOSE SAME ALTITUDES AS COMPUTED ON THE BASIS OF EACH OF SEVEN DIFFERENT VALUES OF Q

	P11	P20	P32	P47	P53	P61	P79
STD	.226320	.547487	.868014	.110905	.590005	.182099	.10377
Q1	.22632071	.54748903	.86801895	.11090634	.59000958	.18210080	.10377124
Q2	.22632066	.54748882	.86801839	.11090626	.59000910	.18210062	.10377110
Q3	.22632055	.54748833	.86801712	.11090602	.59000773	.18210013	.10377072
Q4	.22632046	.54748789	.86801599	.11090580	.59000645	.18209967	.10377038
Q5	.22632037	.54748747	.86801489	.11090559	.59000522	.18209923	.10377003
Q6	.22632030	.54748712	.86801406	.11090546	.59000446	.18209898	.10376985
Q7	.22632026	.54748691	.86801350	.11090534	.59000378	.18209874	.10376967

Notes:

1. A single underline extending under all eight of the digits of a particular entry designates that if this number is rounded to six significant figures the result is in exact agreement with the value tabulated in the standard atmosphere.
2. A single underline extending under only the first six of the digits of a particular entry designates that if this number is chopped off after the first six significant figures the result is in exact agreement with the standard atmosphere value.
3. The existence of both a short and a long underline designates that either a chopping or rounding operation would result in exact agreement with the standard atmosphere value.

Values of pressure underlined (first six significant figures only) would be in complete agreement with the Standard Atmosphere when the computer program is set to chop at the end of six significant figures. Values of pressure for H = 11 km' corresponding to all seven values of Q are so underlined. Any value of Q which yields such underlined values of pressure at consecutive critical altitudes from 11 to 79 km would be a suitable choice for the standard atmosphere recalculation. Only Q5 meets this criterion. Values represented by Q1 and Q2 which one would have expected to be used in the U.S. Standard are seen not to be suitable.

In cases where rounding of the sixth significant figure, in accordance with the value of the seventh and eighth significant figure, would have given the standard-atmosphere value, an underline under all eight digits is used. Thus for P20, values of Q5 through Q7 would have been satisfactory if a rounding procedure had been employed, while only Q4 through Q6 would be satisfactory if the chopping technique had been employed. We see from values of P53 that only Q5 yields pressures which match the standard pressure for that altitude. It is also apparent that Q5 also yields the standard-atmosphere pressures at all other critical altitudes. For P32 and P47, however, Q5 is applicable only when the chopping technique is employed. Thus Q5 is most likely the value employed in the original computation of the 1962 U.S. Standard Atmosphere, and the pressure values appear to have been chopped.

The value of Q5 = 0.03416323 is considerably different from Q = 0.03416319, the most probable seven-significant figure value which one would expect from the values shown in Table 18. It is questioned, therefore, whether the pressure values published in the U.S. Atandard Atmosphere can truly be considered to be correct in terms of the definitions and the values for G, M_0 and R^* specified in that standard. In any event, the actual lumped value of Q employed should have been specified in the Standard Atmosphere along with a statement regarding rounding or cutting of the values of the various properties.

Determination of Pressure at the Critical Altitude of 90,000 Geometric Meters

Having deduced that value of the constant $G\ M_0/R^*$ which permits the duplication of standard-atmosphere pressures at the critical levels between 0 and 79 km inclusive, one should be able to compute the pressure at 90 geometric kilometers, provided that the equivalent value of geopotential altitude is known to a sufficient number of significant figures (since this point is at the upper end of the geopotential regime). The value of H for Z = 90,000 meters exact is given in the Standard Atmosphere to only five-significant figures, i.e., 88743 m'. This value is not satisfactory for computing the reference-level pressure at 90,000 m to six or seven significant figures since a change of three units in the sixth-significant figure of geopotential may produce a change of about one unit in the fifth-significant figure of pressure.

Equation (31) and the pressure value at 79 km$'$ corresponding to Q5 in Table 19, i.e., 1.0377003×10^{-2} mb, provide the means for calculating the pressure at various altitudes in the isothermal layer above 79 km$'$. At 88743.355 m$'$ the geopotential equivalent of an altitude of 90,000 geometric meters, the top of the isothermal layer, the pressure is found to be 1.6437806×10^{-3} mb. This value compares with the 5 significant figure value of 1.6438×10^{-3} mb given for 90,000 m in the tabulations of the Standard Atmosphere, and the value of 1.6439×10^{-3} mb which results when H is taken to be 88743 m$'$. Contrary to the situation for the critical altitudes below 90 km, the tabulated standard atmosphere value of pressure at 90 km appear to have been rounded from the 8 significant figure value given above. This situation indicates that the above considerations may not have completely resolved the problem of the more accurate determination of pressure at Z = 90,000.00 meters. Never the less the value 1.6437806×10^{-3} mb was adopted as the base for further calculation. The value 1.6437806×10^{-3} mb is adopted as the base for further calculation.

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APPENDIX A

GCA Technical Report 65-27-N

CRITICAL EXAMINATION OF EQUATIONS FOR ATMOSPHERIC NUMBER-DENSITY CALCULATIONS — COMPUTED VALUES COMPARED WITH OBSERVATIONS

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ABSTRACT

Some previously published equations for computing particle number density as a function of altitude are shown to have functional limitations which lead to progressively erroneous results as the temperature gradient approaches some realistic non-zero value. Development of series expansions for these equations has eliminated the basic problem. Other number-density equations based on the same fundamental premises are somewhat simpler and are formulated in such a way so as to shift the functional difficulty to the region of zero temperature gradient. Appropriate series expansions of these equations eliminate the functional difficulty and result in expressions which are not only simple, but also converge with great rapidity, so that usable results are obtained from the first term of the series and two terms usually lead to five-significant figure accuracy.

Helium number densities computed by means of such equations for temperature-altitude profiles of two widely differing model atmospheres are presented. A comparison of the computed values with observed data suggest a variation in the onset level of helium diffusive separation of as much as 20 kilometers.

APPENDIX A (Continued)

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APPENDIX B

GCA Technical Report No. 66-4-N

DENSITY-ALTITUDE DATA
FROM 150 ROCKET FLIGHTS AND 26 SEARCHLIGHT PROBINGS, 1947 THROUGH 1964

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Contract No. NASW-1225

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DENSITY-ALTITUDE DATA
FROM 150 ROCKET FLIGHTS AND 26 SEARCHLIGHT PROBINGS, 1947 THROUGH 1964

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SUMMARY

Tabulations of 217 density-altitude profiles measured in the altitude region between about 20 km and 220 km are presented. Three of these profiles consist of a single density-altitude value. One consists of 167 such values. Many of the profiles contain about 50 density-altitude values. These density altitude profiles have been measured at eleven land-based sites and at seven ship sites at sea over a period of about eighteen years, i.e., from March 1947 to February 1965. These tabulations generally do not include the results of the meteorological-rocket-network flights which are published elsewhere. A complete list of references indicates the origin of all the data.

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APPENDIX C

GCA Technical Report No. 66-5-N

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A STATUS REPORT ON ATMOSPHERIC DENSITY MODELS AND OBSERVATIONS

R. A. Minzner

Contract No. NASW-1225

February 1966

GCA CORPORATION
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Bedford, Massachusetts

Prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Headquarters
Washington, D.C.

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A STATUS REPORT ON ATMOSPHERIC DENSITY MODELS AND OBSERVATIONS

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SUMMARY

The history of standard and model atmospheres reflects the political as well as the scientific thinking of the related eras, and follows a pattern of successively extending the upper-altitude limit of the atmospheric model in keeping with the successive increases in the altitude range of the human traveler or his machines and measuring equipment. The upward extensions of the atmospheric models have frequently been based on speculation or, at best, upon limited knowledge; and as improved information became available the need for revisions of these models became evident. Frequently, the refinements in measuring techniques which made possible the extension at the high-altitude end of the model also provided the basis for improvements and modification of the intermediate-altitude regions which were then defined in a quasi-legal manner to represent a best average model.

The vast amount of satellite drag-acceleration data acquired from the measured orbits of artificial earth satellites above 200-km altitude over the past nine years has led to the recognition that at high altitudes, at least, the atmosphere varies greatly with time; that is, from day to night as well as from periods of high solar activity to periods of low solar activity. Several models reflecting these variations have been developed, and these indicate a shift in thinking away from the concept of a single average model to that of the multiple model showing variability. During this same nine-year period the number of observations of atmospheric properties below 120-km altitude has also increased considerably, though not nearly at the same rate as those above 200 km. Models showing seasonal and latitudinal variations below 120 km have been prepared — some on the basis of rather limited data. These models suggest an isopycnic region at about 90 km with a density about 14 percent greater than that of the 1962 United States Standard Atmosphere at that altitude. These models also suggest seasonal variability to be minimum at tropical latitudes and to increase to maximum at sub-polar latitudes. No diurnal variation has yet been suggested at altitudes below 120 km.

A re-examination by the writer of seasonal and latitudinal variations of atmospheric density on the basis of 209 density-altitude profiles between 30 and 200 km (not including data gathered in the Meteorological Rocket Network) is currently under way. These data include the results from seven different measuring techniques employed at eleven different land sites and seven different sea sites. Preliminary results indicate that the mean summer and mean winter density-altitude profiles for 30°N latitude exhibits a crossing of isopycnic layer near 90-km altitude. A similar situation exists for mean-summer and

mean-winter density-altitude profiles for 38^{o}N as well as for the pair of profiles for 58^{o}N, but each occurs at a considerably different value of densi The 38^{o} data and the 58^{o} data each exhibit an additional isopycnic level abou 2 scale heights above the near-90-km isopycnic level. The existence of this additional isopycnic level is in keeping with the predictions of a simple theory.

The mean of all data shows the standard-atmosphere densities to be too l between 83 km and some altitude above 120 km with the discrepancy being in excess of 40 percent in the vicinity of 95 km. Percentage departure-versus-altitude profiles of the set of recently adopted United States Supplementary Atmospheres appear to be increasingly in conflict with the data for measuring altitudes above 90 or 95 km,particularly for 38^{o}N winter and for 58^{o}N winter summer. More data in the 100 to 200 km region are urgently needed.

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APPENDIX D

GCA Technical Report No. 66-6-N

TEMPERATURE DETERMINATION OF PLANETARY ATMOSPHERES

OPTIMUM BOUNDARY CONDITIONS FOR BOTH LOW AND HIGH SOLAR ACTIVITY

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Contract No. NASW-1225

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APPENDIX D (Continued)

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SUMMARY

Accurate temperature-altitude profiles of planetary atmospheres for altitudes above the diffusive separation level may be determined from the simultaneously observed altitude profiles of the number density of two inert gases having markedly different molecular weights M, without any assumptions concerning reference-level temperatures. In the earth's atmosphere, such gases would preferably be helium and argon. If the altitude profile of the number density of but a single inert heavy gas is measured, the temperature-altitude profile is obtainable only for the lower portion of a sufficiently large altitude interval of observation by means of the downward application of a single-gas equation. If, on the other hand, the observed number-density data are for a single light gas such as helium, a temperature-altitude profile may be determined by an upward application of the single-gas equation, but only if the temperature is independently known at the lowest altitude of observed number-density data. If both the light and heavy gases are measured simultaneously these two sets of number desities introduced into a dual-gas equation permit the determination of the temperature over the entire altitude interval of the dual-gas observation. Previously described methods using the mass-density profile or the total number-density profile in a mixed or diffusively separating atmosphere permitted the determination of only the ratio of temperature to mean molecular weight and that over only a limited portion of the altitude interval of the observed data. In contrast, to this situation the one-gas and two-gas methods described yield values of kinetic temperature directly. In the two-gas method the temperatures is determined not only at the lower altitudes where both heavy-gas and light-gas data may be measured but also up to the highest altitudes for which the light-gas number density has been measured, but where the heavy-gas number density has decreased to values below the detection sensitivity of the sensor.

An analytical and numerical examination of the single-gas and double-gas equations for both upward and downward calculations (that is, for both high-altitude and 10w-altitude reference levels) using atmospheric models for both high and low solar activity, indicates the conditions which optimize each type of calculation. The method depends upon recently developed air-borne mass spectrometers with detection sensitivities of the order of 10^4 to 10^5 particles per cubic centimeter.

An error analysis based on the gaussian method has been applied to the various temperature equations where the perfect integral of number density over a specified altitude interval has been approximated by a numerical-integration expression developed from a logarithmic tropezoidal rule. Number-density uncertainties based upon the sensitivities of present-day mass spectrometers were used in the numerical evaluation of the error expressions. The error analysis demonstrates that two sets of single-gas data applied consecutively and iteratively for a number of cycles to two appropriate single-gas equations yield temperature uncertainties which are essentially indentical to those obtained by the single application of each of two appropriate double-gas equations. The error analysis further demonstrates: (1) That for optimum conditions,

the high-altitude reference level should be chosen as the altitude for which the uncertainty in observed number-density data is 100 percent. (2) That the absolute temperature uncertainty is not strongly influenced by variations in number-density models associated with variations in solar activity. (3) That the percentage uncertainty at high altitudes, however, is strongly influenced by such variations in number-density models.

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